Advanced Modeling & Simulation (AMS) Seminar Series
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High-order discontinuous Galerkin method for entry simulations into dusty atmospheric environments

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Objective

Summary of research results from NASA-supported ECF award NNX15AU58G on "Advanced Physical Models and Numerical Algorithms to Enable High-Fidelity Aerothermodynamic Simulations of Planetary Entry Vehicles on Emerging Distributed Heterogeneous Computing Architectures"

Overall research goals

- Develop innovative physical models and advanced numerical methods for reliably predicting aerothermodynamic flows that are relevant to hypersonic and atmospheric entry vehicles in dusty flow environments

Objective

- Develop high-order DG solver for high-speed disperse multiphase flows
  - Shock capturing method, with a focus on predicting surface heating in hypersonic flows
  - Lagrangian particle method suited for arbitrary curved, high-aspect-ratio elements
- Assess DG method for hypersonic, particle-laden flow applications
  - Quantitative comparisons with state-of-the-art finite volume solvers (FUN3D, LAURA)
- Apply developed DG framework to hypersonic dusty flows simulations
  - Parametric sensitivity study
  - Hypersonic dusty flow over the ExoMars Schiaparelli capsule

Acknowledgments

NASA

- Michael Barnhardt, Grant Palmer, Khalil Bensassi, Peter Gnoffo, Ethiraj Venkatapathy, Amal Sahai, and entire EDL-team

DLR

- Ali Gülhan, Dirk Allofs

Stanford

- Yu Lv, Steven Brill, Narendra Singh, Alex Aiken

Outline

Background → **Development** → **Application**

Background
- Introduction & Motivation
- DG mathematical background

Development
- Shock capturing scheme
- Lagrangian particle method

Application

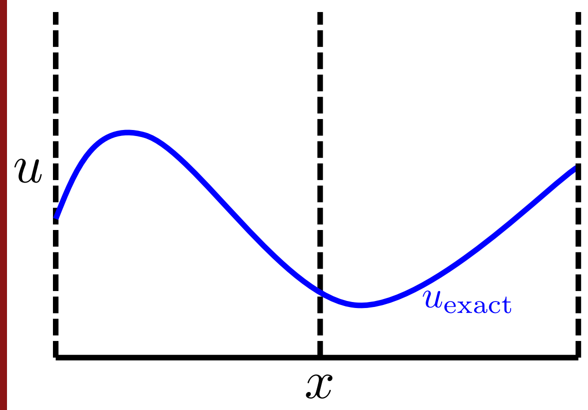
Hypersonic dusty flows over blunt bodies
- Experimental data and parametric sensitivities
- Mars atmospheric entry

What is DG?

Discontinuous Galerkin methods are a family of numerical methods combining features of finite-element and finite-volume schemes

- Piecewise polynomials used to approximate the solution
  - Arbitrary polynomial order p
  - In each element, solve for polynomial coefficients $\tilde{u}$

$$u_{\text{approx}}(t,x) = \sum_{i=1}^{p+1} \tilde{u}_i(t)\phi_i(x)$$



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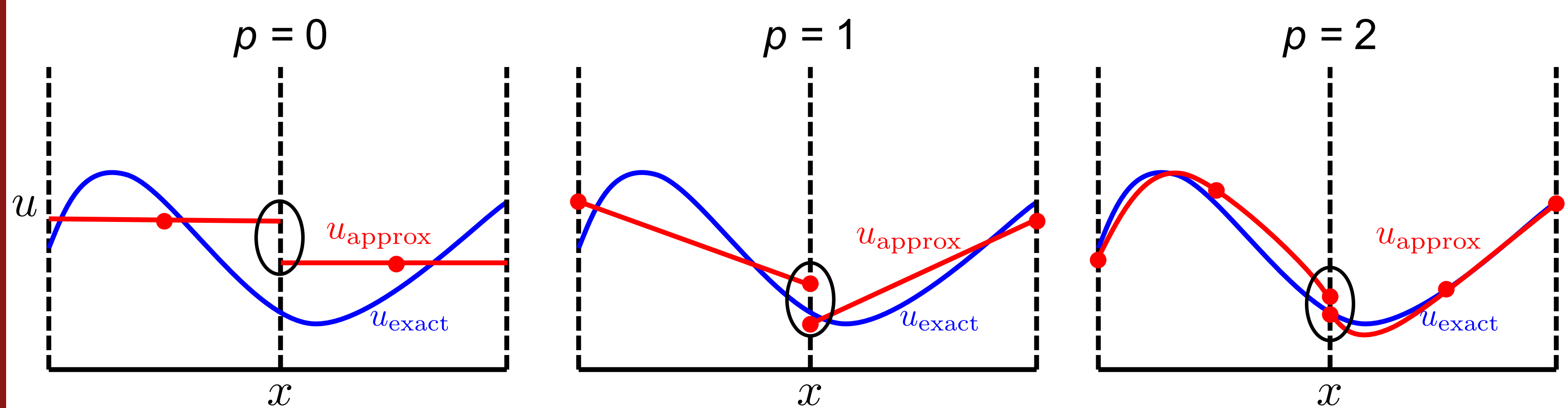
- Numerical fluxes needed to deal with discontinuities between elements



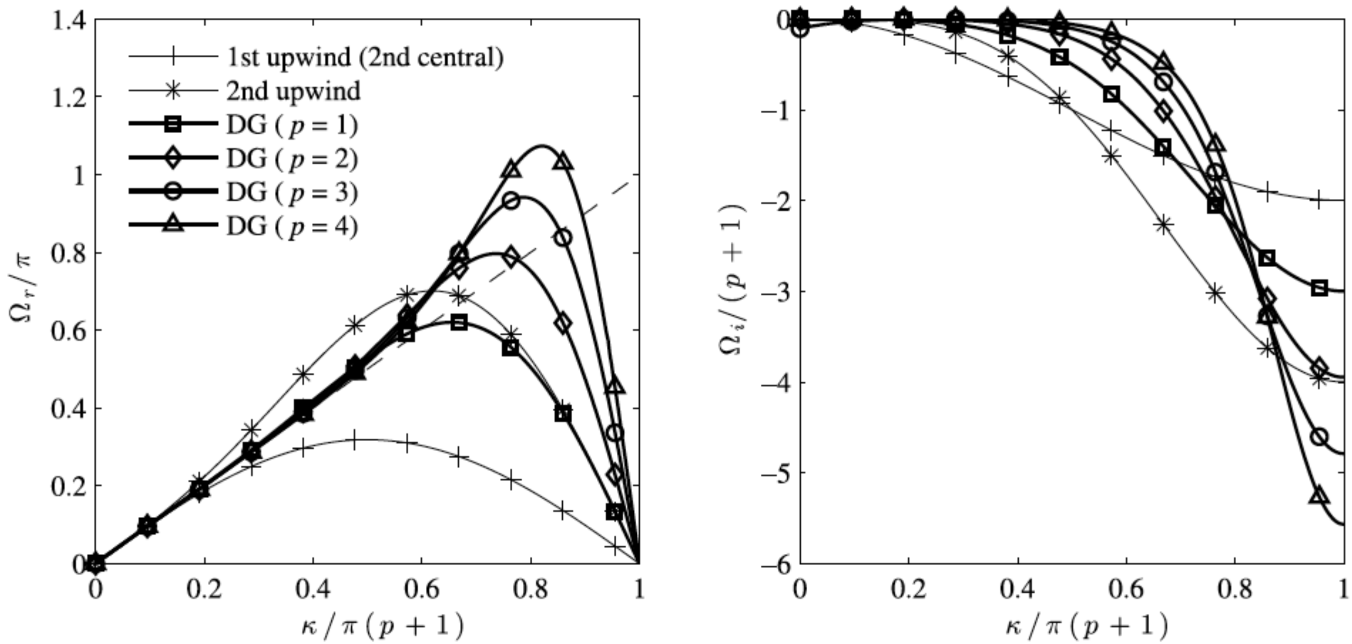
Why DG?

Current state of simulations of multi-physics (turbulent, multiphase, chemically reacting, etc.) flows on complex geometries

- Finite-difference, finite-volume methods
- Low-order (1st or 2nd order)

Potential for high-order DG methods

- Desirable dissipation and dispersion properties



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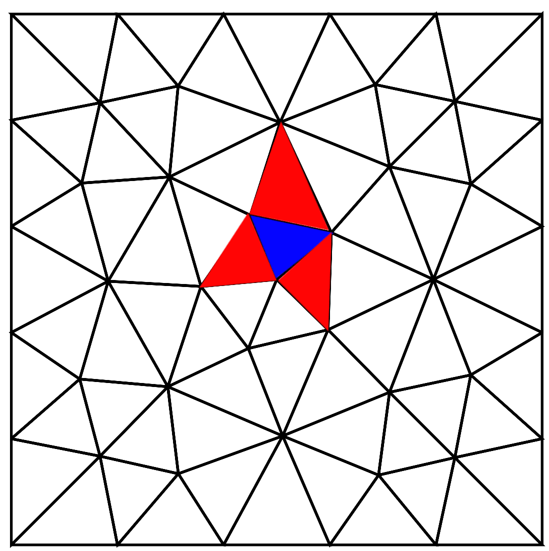

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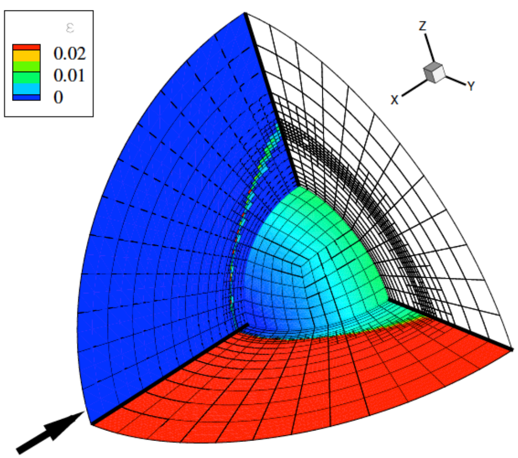
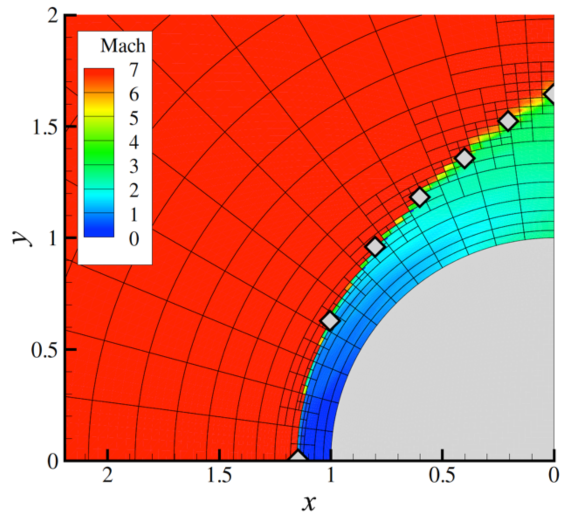
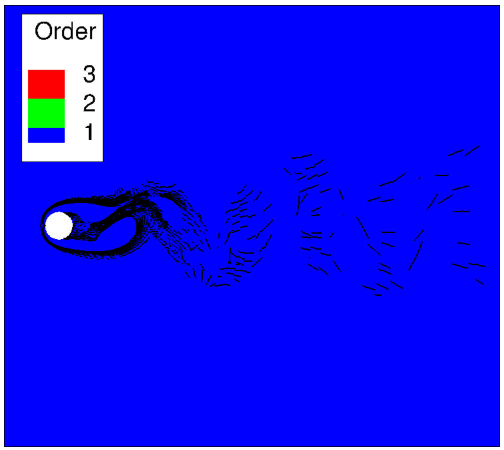
Potential for high-order DG methods

- Desirable dissipation and dispersion properties
- Suited for HPC (compactness, FLOPs : memory bandwidth)
- Local mesh-adaptation (*h*) and refinement in polynomial order (*p*)

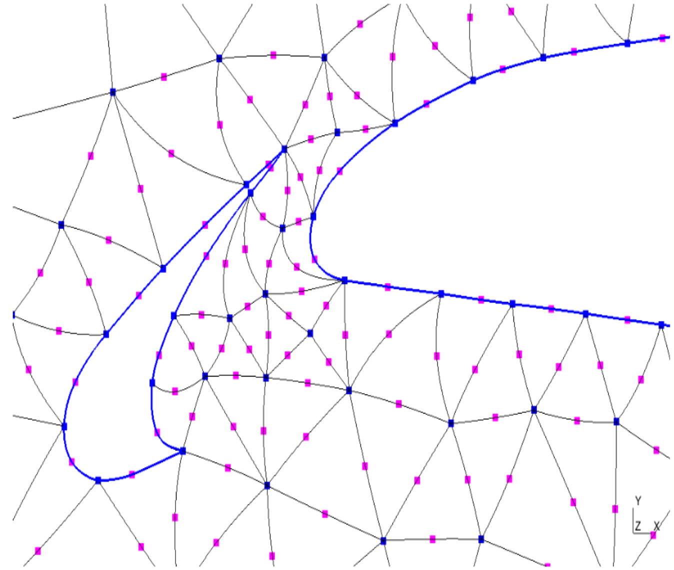


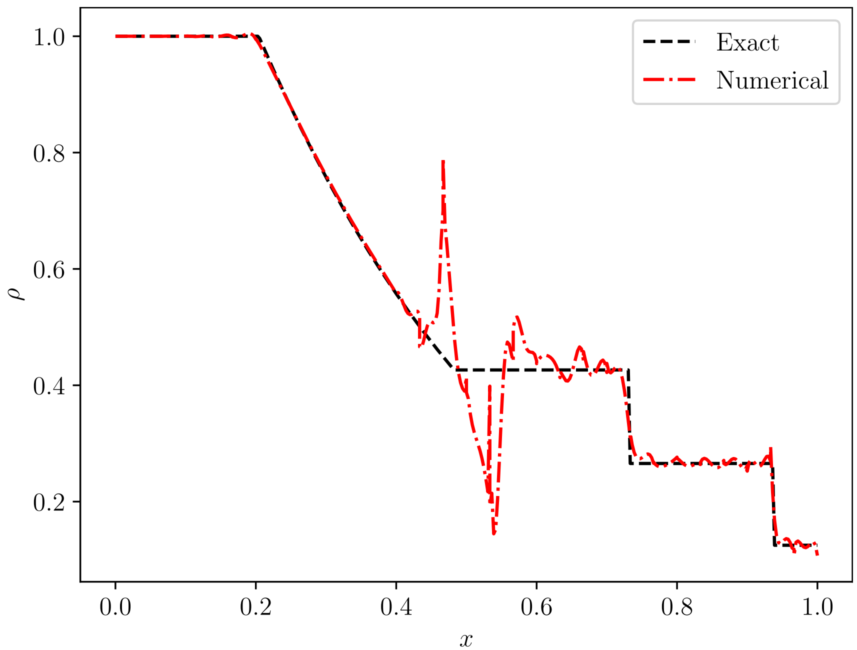


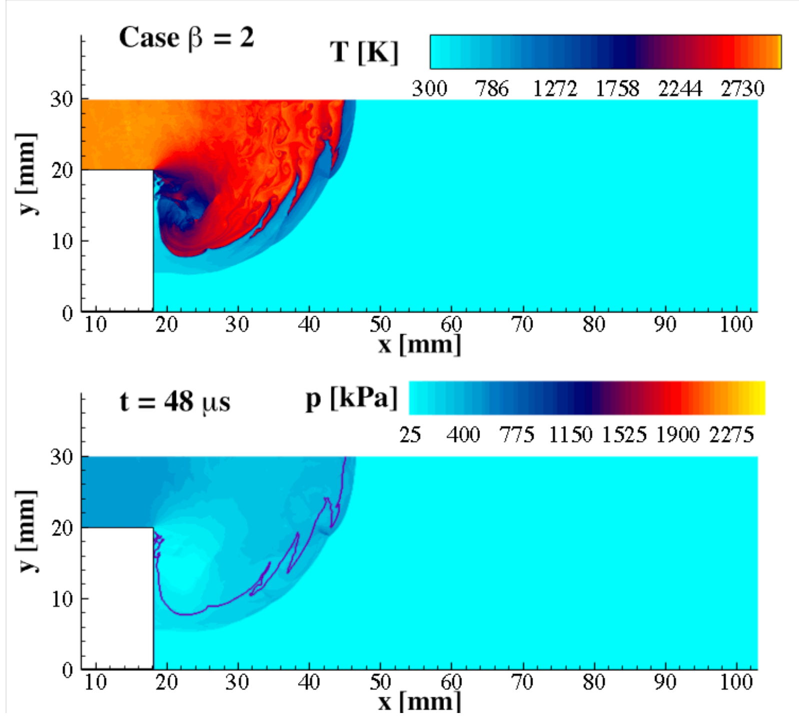


Challenges

- Efficient time stepping
- Efficient automatic *hp*-adaptation
- High-order curved meshes
- Numerical instabilities
- Extensions to complex physics







[1] Lv and Ihme, *Proc Combust Inst*, 2015

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- Experimental data and parametric sensitivities
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Discontinuous Galerkin discretization

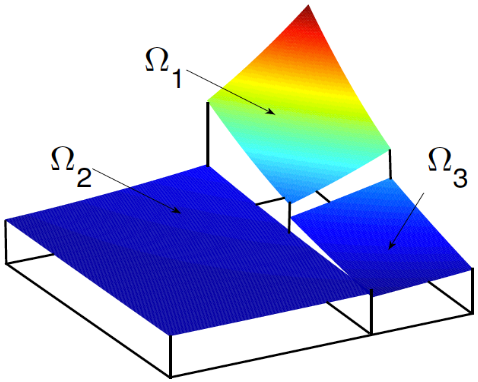
Governing equations

$$\partial_t \boldsymbol{U} + \nabla \cdot \boldsymbol{F}_s = \nabla \cdot \boldsymbol{F}_v + \boldsymbol{S}$$

$$\boldsymbol{U} = \begin{bmatrix} \rho \\ \rho\boldsymbol{u} \\ \rho E \end{bmatrix}, \quad \boldsymbol{F}_s = \begin{bmatrix} \rho\boldsymbol{u} \\ \rho\boldsymbol{u}\otimes\boldsymbol{u} + P\mathbb{I} \\ \boldsymbol{u}(\rho E + P) \end{bmatrix}, \quad \boldsymbol{F}_v = \begin{bmatrix} 0 \\ \boldsymbol{\tau} \\ \boldsymbol{u}\cdot\boldsymbol{\tau} - \boldsymbol{q} \end{bmatrix}$$

Partition computational domain

$$\Omega = \bigcup_{e=1}^{N_e} \Omega_e$$



Approximate solution with polynomials of order *p*

$$\boldsymbol{U} \approx \boldsymbol{U}_h = \oplus_{e=1}^{N_e} \boldsymbol{U}_h^e$$

$$\boldsymbol{U}_h^e(\boldsymbol{x}, t) = \sum_{n=1}^{N_b} \widetilde{\boldsymbol{U}}_n^e(t)\phi_n(\boldsymbol{x})$$

Multiply governing equations by ϕ_m and integrate

Discontinuous Galerkin discretization

Find the basis coefficients $\widetilde{\boldsymbol{U}}^{e}(t)$ that discretely satisfy

$$\sum_{n=1}^{N_b} d_t \widetilde{\boldsymbol{U}}^{e}_{n}(t) \int_{\Omega_e} \phi_m \phi_n d\Omega_e + \underbrace{\int_{\Omega_e} \phi_m \nabla \cdot \boldsymbol{F}_s d\Omega_e}_{} = \underbrace{\int_{\Omega_e} \phi_m \nabla \cdot \boldsymbol{F}_v d\Omega_e}_{} + \int_{\Omega_e} \phi_m \boldsymbol{S} d\Omega_e \; \forall \; \phi_m$$

$$m = 1, \ldots, N_b$$

- HLLC[1]
- Roe[2]
- AUSM+[3]
- SLAU[4]

- Interior Penalty[5]
- BR2[6]

Evaluate integrals using Gaussian quadrature

[1] Toro, et al., SW, 1994;
[2] Roe, JCP, 1981;
[3] Liou, JCP, 1996;
[4] Shima and Kitamura, AIAA J., 2011;
[5] Arnold, et al. SIAM JNA, 1981;
[6] Bassi & Rebay, JCP, 1997.

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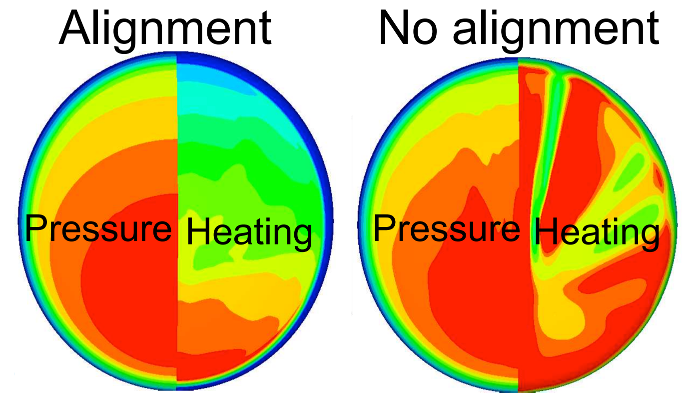
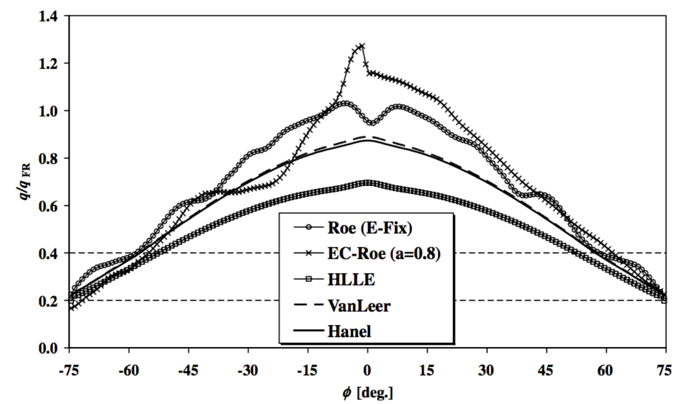
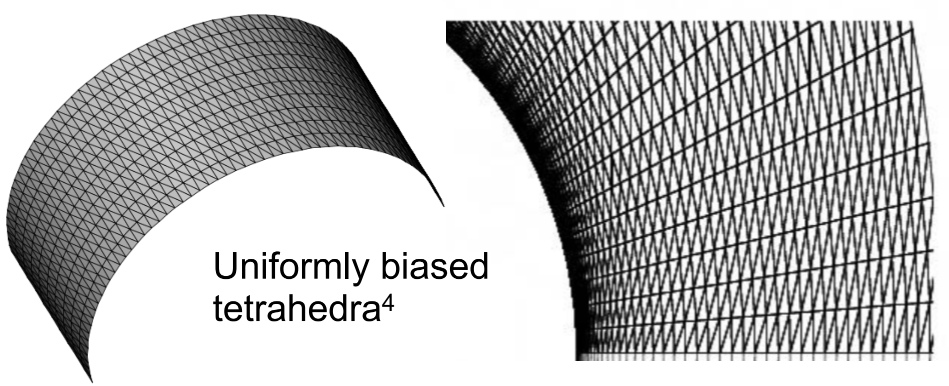
Background: finite-volume

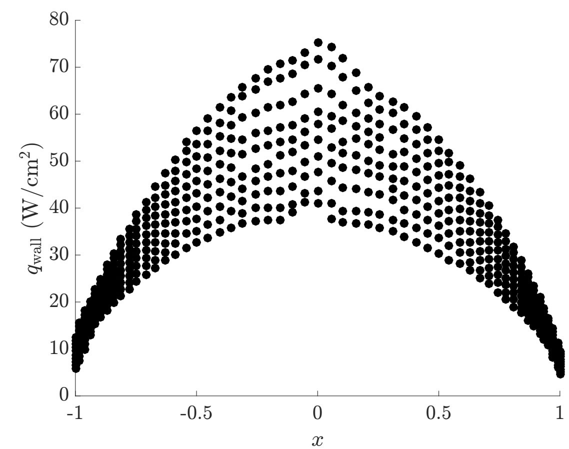
- Surface heating predictions extremely sensitive to:
  - Mesh-shock alignment[1]
  - Inviscid flux function[2,3]
  - Limiter, parameters, etc.[2,3]
- Gnoffo computed hypersonic flow over a cylinder using uniformly biased tetrahedra[4]







Uniformly biased tetrahedra[4]



[1] Candler et al., AIAA Paper, 2009
[2] Kitamura et al., AIAA Journal, 2010
[3] Kitamura, AIAA Paper, 2013
[4] Gnoffo and White, AIAA Paper, 2004

Background: DG

- Robust shock capturing is still an active area of research
- Limited work on using high-order DG to compute viscous hypersonic flows[1,2,3,4]

Goals:

- Develop a simple and robust shock capturing method
- Assess sensitivities of DG heating predictions to mesh-shock alignment and choice of inviscid and viscous flux functions
- Perform quantitative comparisons with FUN3D and LAURA

[1] Barter and Darmofal, *JCP*, 2010
[2] Brazell and Mavriplis, AIAA Paper, 2013
[3] Kercher et al., *IJNMF*, 2020
[4] Fernandez et al., AIAA Paper, 2018

Shock capturing method

Two ingredients

$\boxed{\text{Shock detector}}$

Stabilization mechanism

$$S_e = \sqrt{\frac{\int_{\Omega_e} (\mathsf{s}/\bar{\mathsf{s}} - 1)^2 d\Omega}{|\Omega_e|}}$$

Intra-element variations

- Simple
- Compact
- Large near discontinuities, small in smooth regions

Identify **troubled elements**

[1] Ching et al., *JCP*, 2019

Shock capturing method

Two ingredients

Shock detector

$\boxed{\text{Stabilization mechanism}}$

Artificial viscosity (AV) term $\nabla \cdot \boldsymbol{F}_{\mathrm{AV}}$

$$\boldsymbol{F}_{\mathrm{AV}} = \boldsymbol{H} : \nabla \boldsymbol{U}$$

$$\boldsymbol{H}_{ik} = \eta_{\mathrm{AV}} \frac{h_{ik}}{\bar{h}} \mathbb{I}$$

Add initially elementwise-constant AV to troubled elements

$$\eta^{e}_{\mathrm{AV},0} = C_{\eta} \lambda_{\max} \frac{\bar{h}}{\max(1,p)} \bar{S}_{e}$$

[1] Ching et al., *JCP*, 2019

Shock capturing method

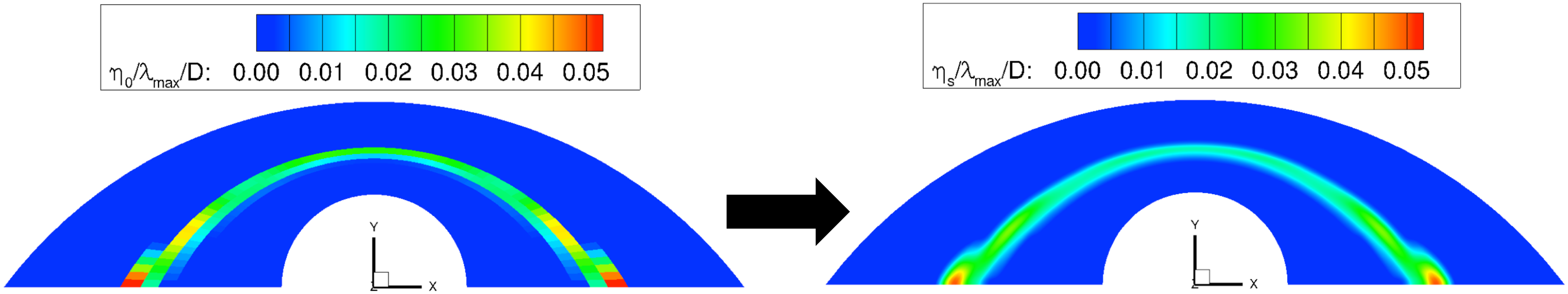
Two ingredients

Shock detector

Stabilization mechanism

Smooth AV

$$\boxed{\eta_{\mathrm{AV},s}} - \nabla \cdot \left[(C_\Delta \Delta)^2 \nabla \boxed{\eta_{\mathrm{AV},s}} \right] = \eta_{\mathrm{AV},0}$$



Reduce AV in boundary layers

- Homogeneous Dirichlet BCs at no-slip walls
- Pass $\eta_{\mathrm{AV},s}$ through sinusoidal filter[2] to get η_{AV}

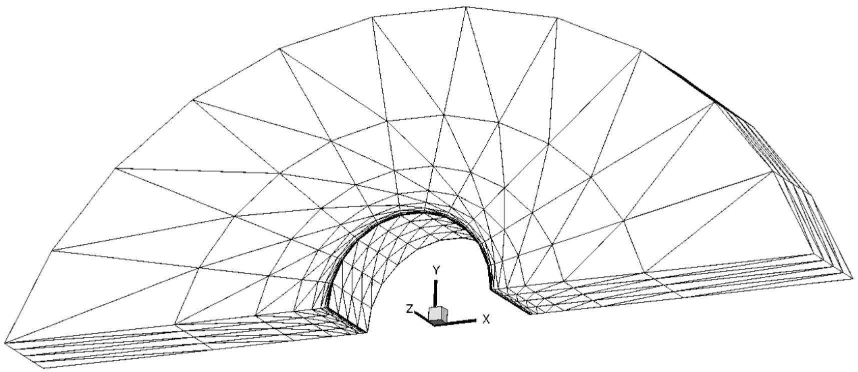
[1] Ching et al., *JCP*, 2019

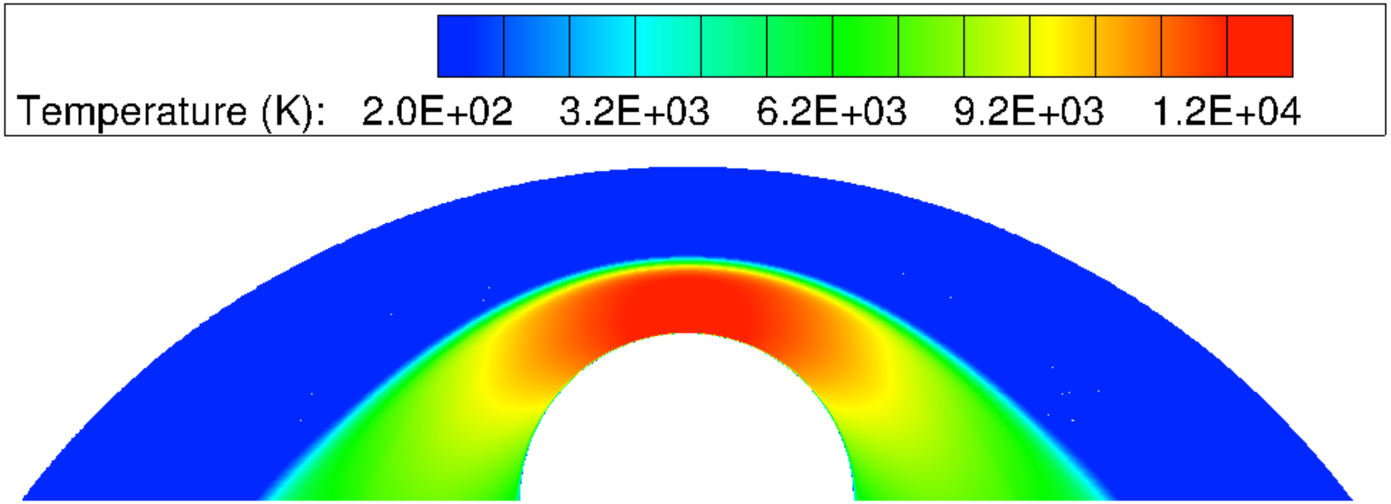
[2] Persson and Peraire, AIAA Paper, 2006

Test case: Mach 17.6 flow over circular half-cylinder

Uniformly biased tetrahedral mesh

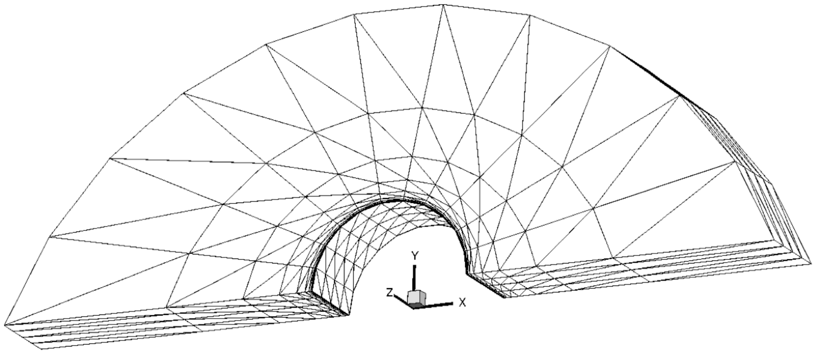
- DG: (p+1)-order accuracy, where p = 1, 2, 3
- Quantitative comparisons with FUN3D
  - 2nd-order accuracy
- Influences of inviscid and viscous flux functions on heating predictions



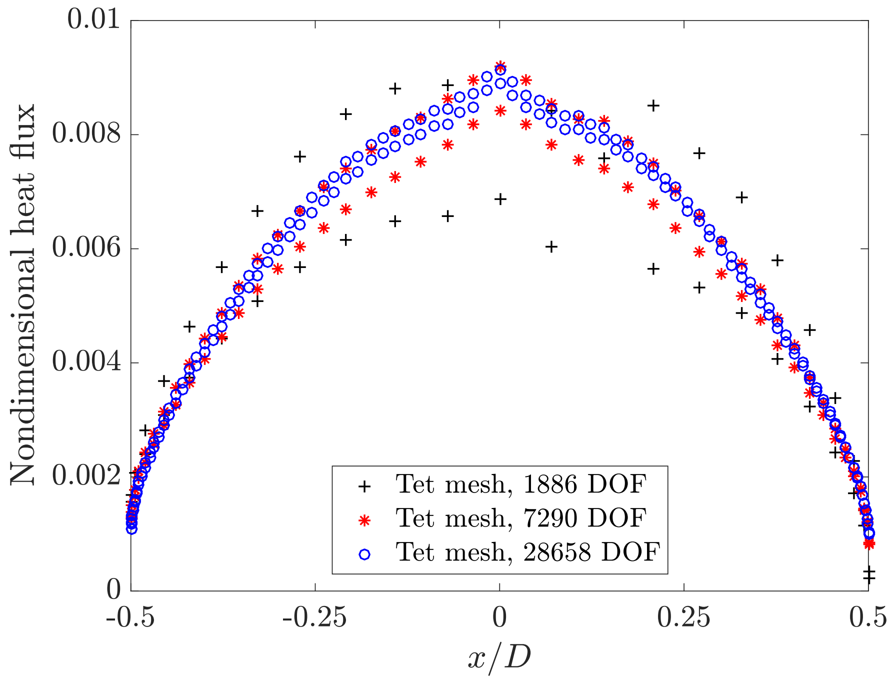


Ma	*p*-order
17.6	1-3

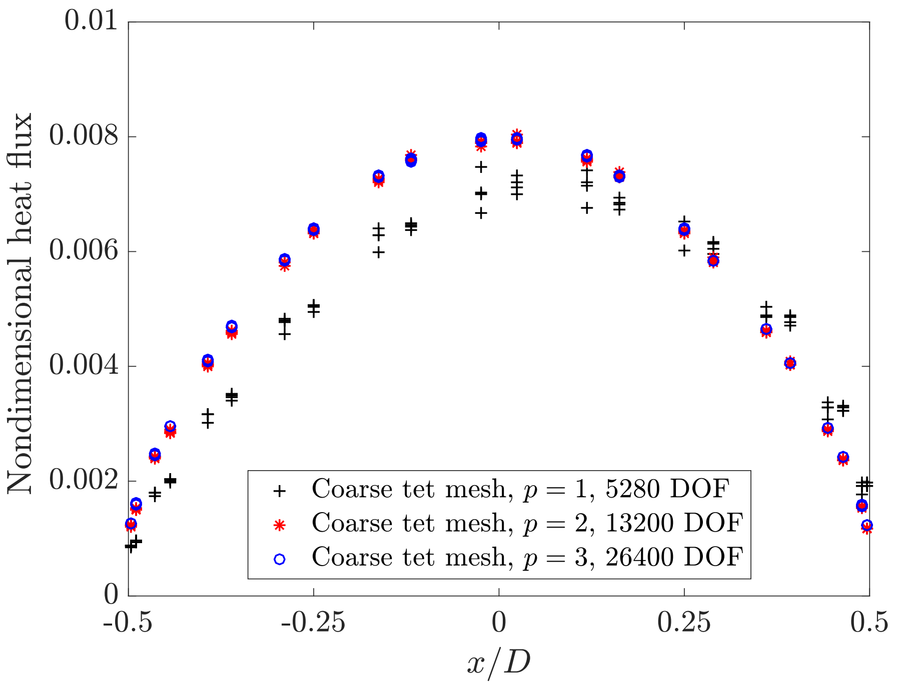
FUN3D heat flux comparisons



FUN3D

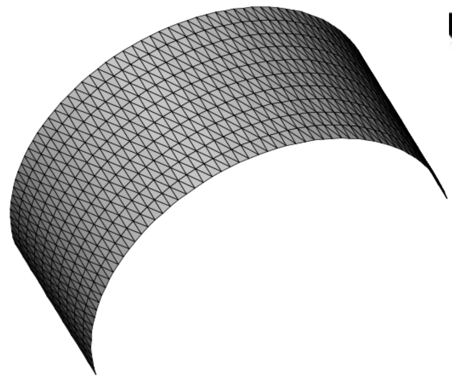


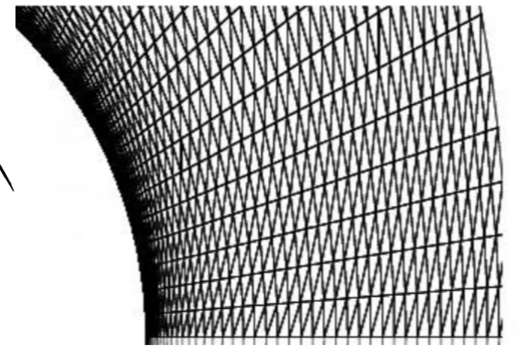
DG



Ma	*p*-order
17.6	1-3

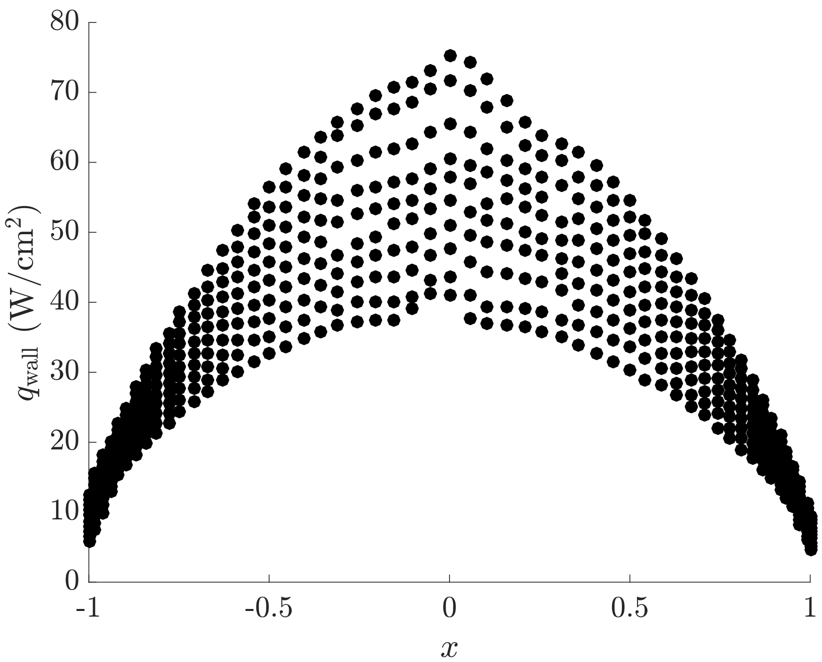
FUN3D heat flux comparisons

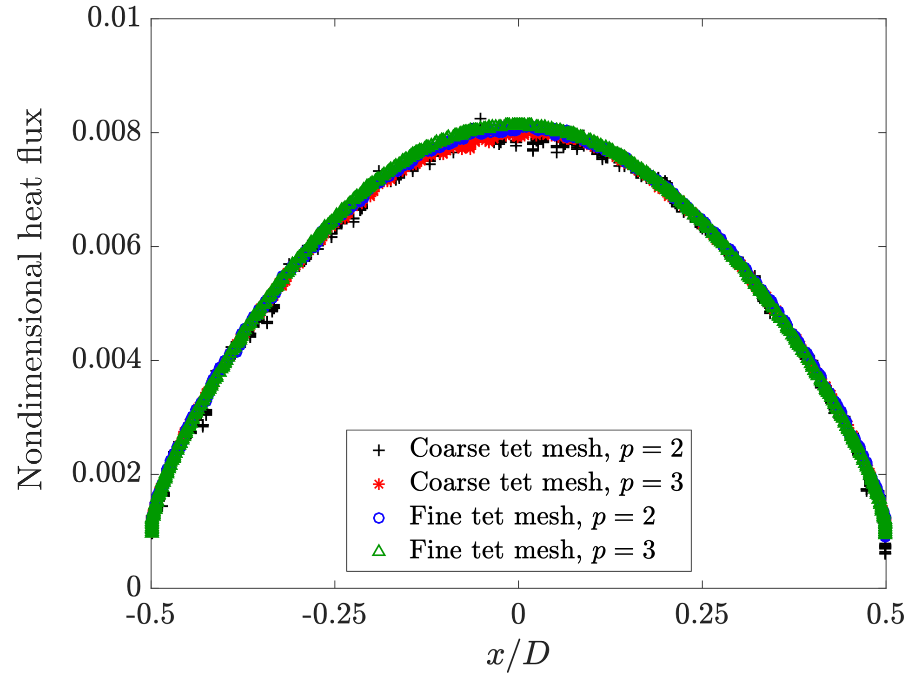




FUN3D

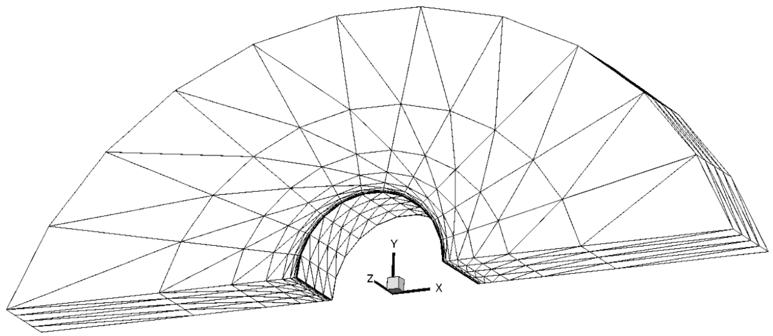
DG



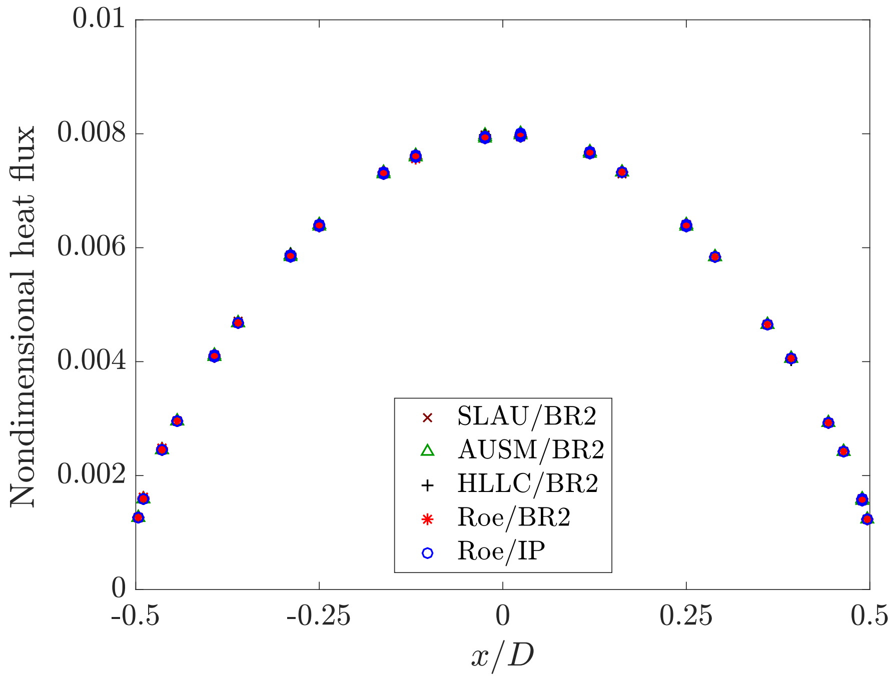


Ma	p-order
17.6	3

Sensitivity to inviscid and viscous flux functions

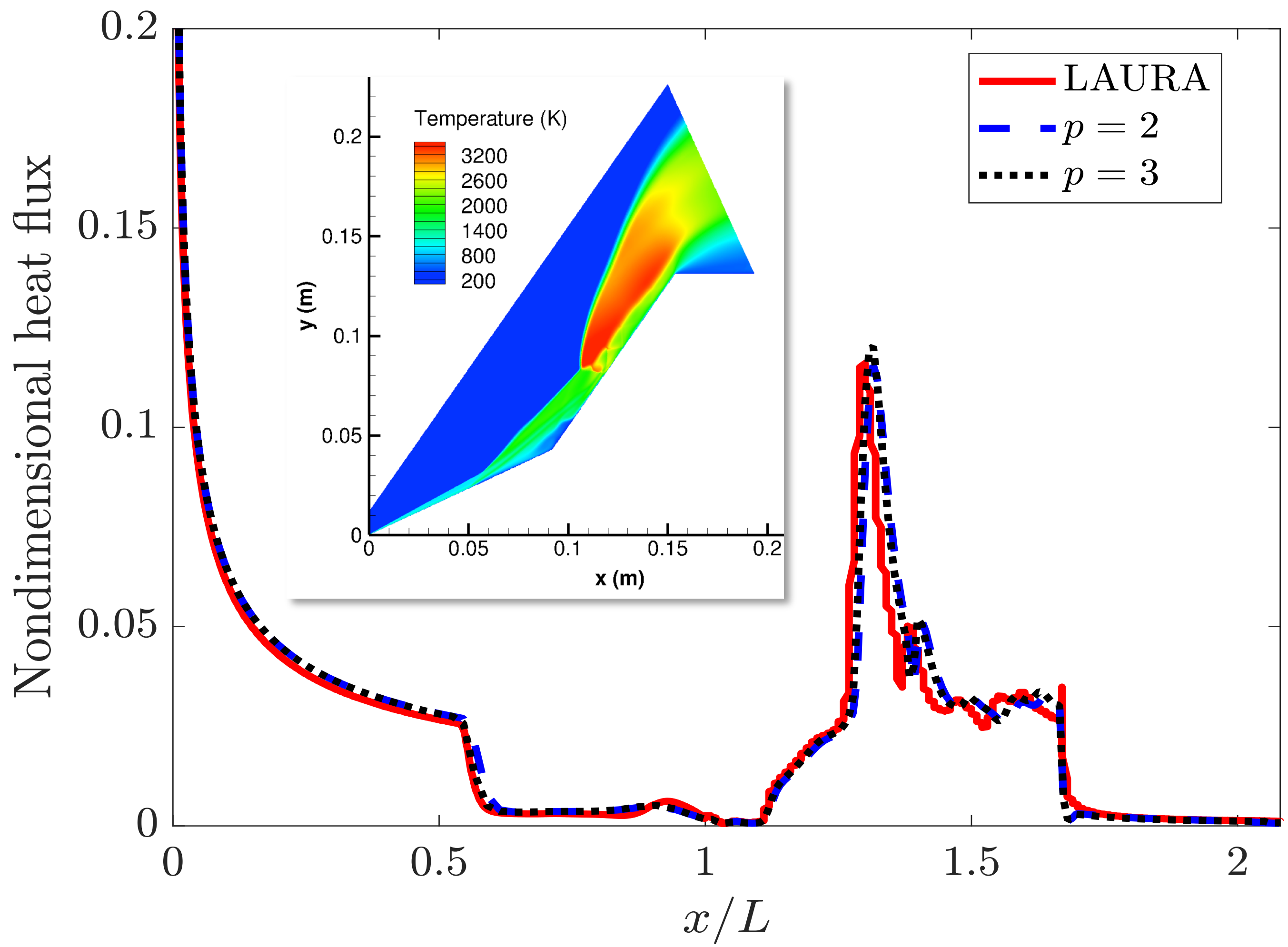


Coarse tet mesh, p = 3



Ma	p-order
9.59	2-3

Test case: hypersonic flow over double cone



➔ DG solution reaches convergence with ~half # DOF of LAURA solution

Summary

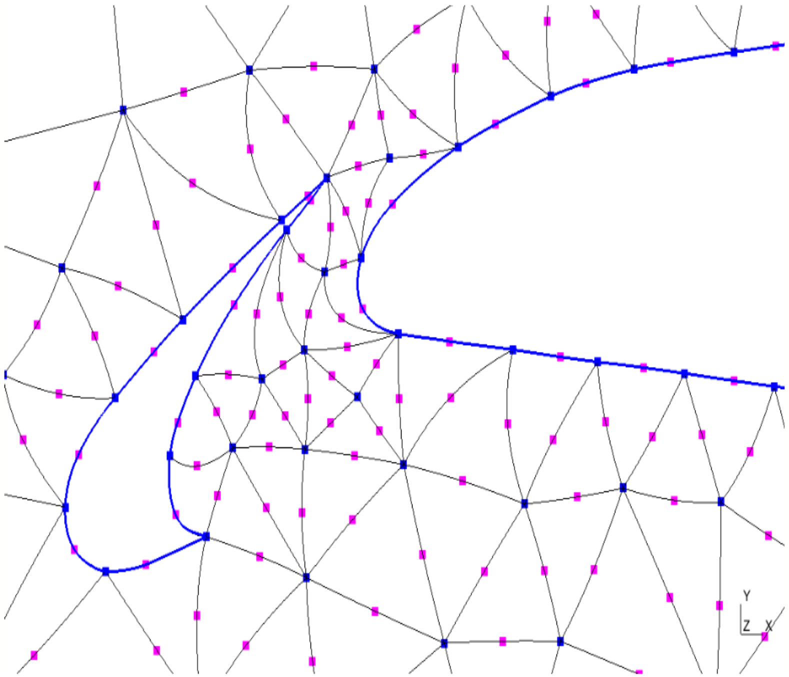
- Developed a simple and robust robust shock capturing method for DG
- Applied the proposed formulation to viscous hypersonic flows
- DG heating predictions are much less sensitive to mesh-shock alignment and choice of inviscid flux function than FV heating predictions
- Fewer degrees of freedom are required to achieve mesh convergence in DG solutions

Outline

Background	Development	Application
▪ Introduction & Motivation ▪ DG mathematical background	▪ Shock capturing scheme ▪ Lagrangian particle method	Hypersonic dusty flows over blunt bodies ▪ Experimental data and parametric sensitivities ▪ Mars atmospheric entry

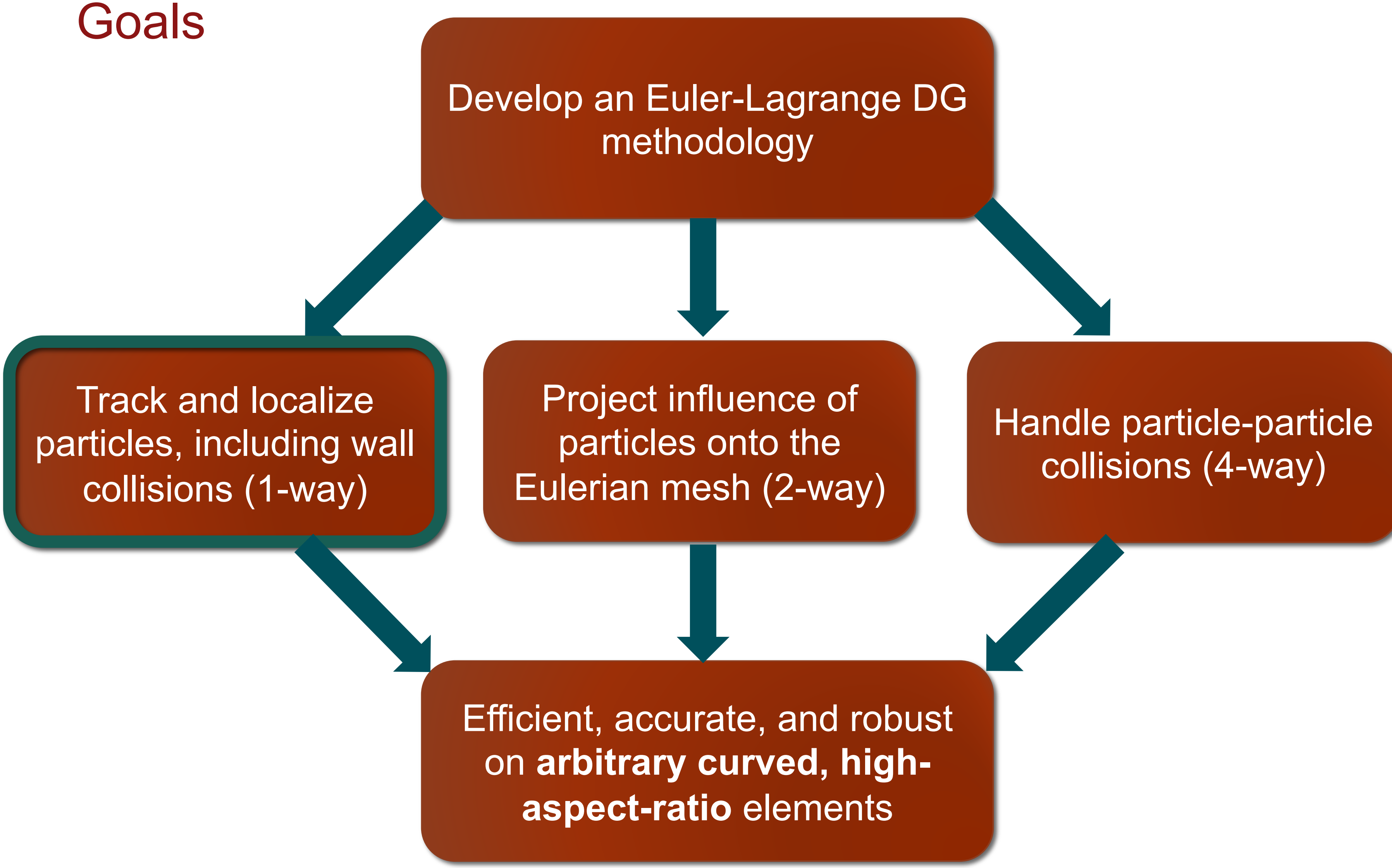
Background: particle-laden flows

- Several point-particle Euler-Lagrange formulations that rely on high-order numerical methods
  - Finite-difference[1,2]
  - Spectral methods[3,4]
- Limited development in the context of:
  - DG[5,6,7]
  - Curved elements



[1] Jacobs and Don, *J Comp Phys*, 2008.
[2] Jacobs et al., *Theor Comp Fluid Dyn*, 2012
[3] Bagchi and Balachandar, *J Fluid Mech*, 2002
[4] Akiki et al., *J Comp Phys*, 2017
[5] Sengupta et al., *Int J Multiph Flow*, 2009
[6] Huang et al, *Comput Methods Appl Mech Eng*, 2019
[7] Zwick and Balachandar, *Int J High Perform Comput Appl*, 2020

Goals


Develop an Euler-Lagrange DG methodology
Track and localize particles, including wall collisions (1-way)
Project influence of particles onto the Eulerian mesh (2-way)
Handle particle-particle collisions (4-way)
Efficient, accurate, and robust on **arbitrary curved, high-aspect-ratio** elements

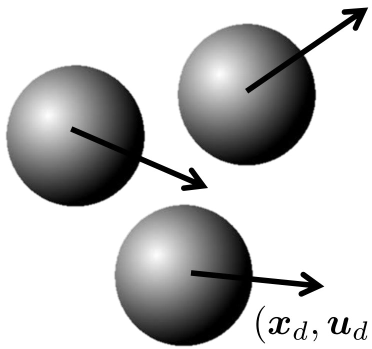
Lagrangian Particle Tracking

- Individual particles ("d") are tracked in the flow field as point sources
- Carrier gas ("c") described with Eulerian field

Position: $$\frac{d\boldsymbol{x}_d}{dt} = \boldsymbol{u}_d$$

Momentum: $$m_d \frac{d\boldsymbol{u}_d}{dt} = \boldsymbol{F} = \boldsymbol{F}_{qs} + \boldsymbol{F}_{\text{other}}$$

Energy: $$m_d c_d \frac{dT_d}{dt} = Q = Q_{qs} + Q_{\text{other}}$$



- Quasi-steady contributions

Drag: $$\boldsymbol{F}_{qs} = \frac{1}{8}\pi D^2 \rho_c (\boldsymbol{u}_c - \boldsymbol{u}_d)|\boldsymbol{u}_c - \boldsymbol{u}_d| C_D$$

Heating Rate: $$Q_{qs} = \pi D \kappa_c (T_c - T_d)\mathrm{Nu}$$

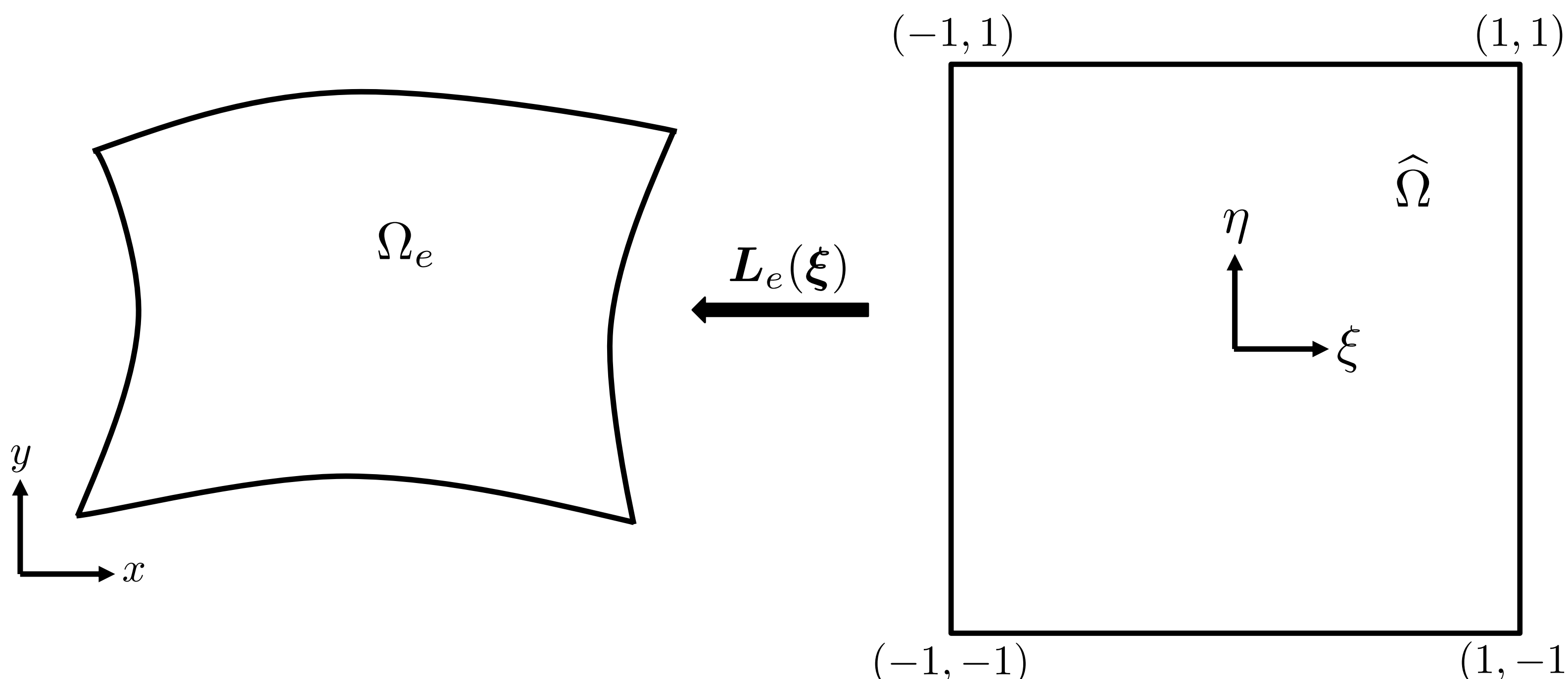
$$C_D = f(\mathrm{Re}_d, \mathrm{Ma}_d) \qquad \mathrm{Nu} = f(\mathrm{Re}_d, \mathrm{Ma}_d)$$

Particle search-locate procedure

Algorithm by Allievi and Bermejo

- Compatible with **arbitrary curved elements**
- Relies on the geometric mapping

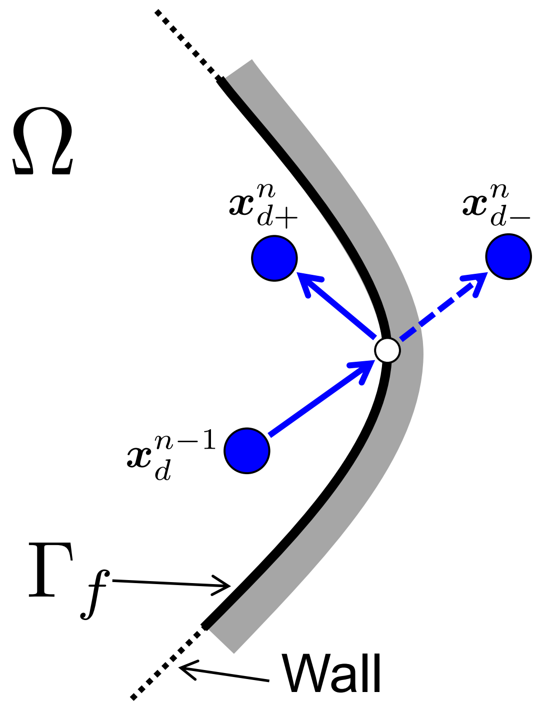
$$\boldsymbol{L}_e(\boldsymbol{\xi}) \equiv \begin{bmatrix} x \\ y \end{bmatrix} = \sum_{i=1}^{N_n} \psi_i(\boldsymbol{\xi}) \begin{bmatrix} x_i^e \\ y_i^e \end{bmatrix}$$



[1] Allievi and Bermejo, *JCP*, 1997

Particle-wall collisions

- Extended the search-locate procedure to account for hard-sphere particle-wall collisions
- Apply Newton search to compute intersection between particle trajectory and boundary
- **Curved, high-aspect-ratio** elements
  - Pathological cases[1]



[1] Ching et al., *JCP*, 2020

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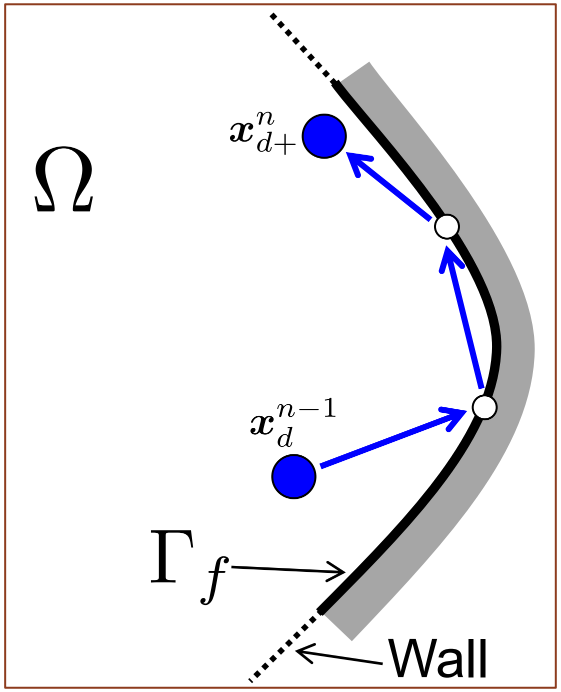
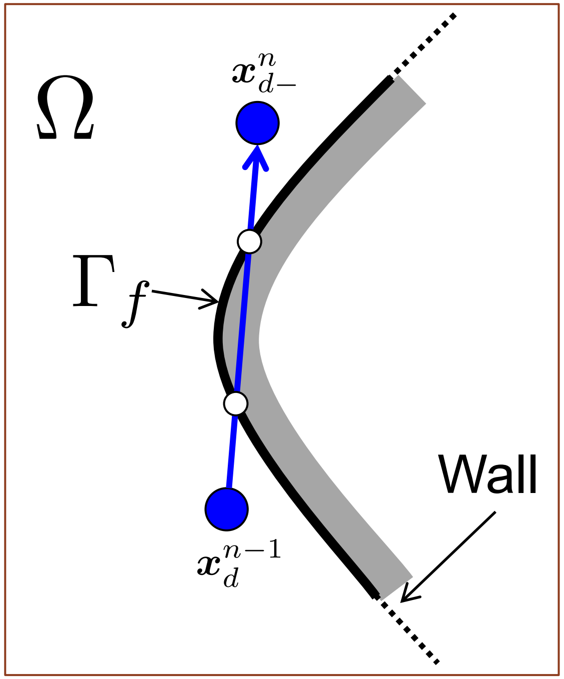
- Extended the search-locate procedure to account for hard-sphere particle-wall collisions
- Apply Newton search to compute intersection between particle trajectory and boundary
- **Curved, high-aspect-ratio** elements
  - Pathological cases[1]
  - Finite size of particles[2]

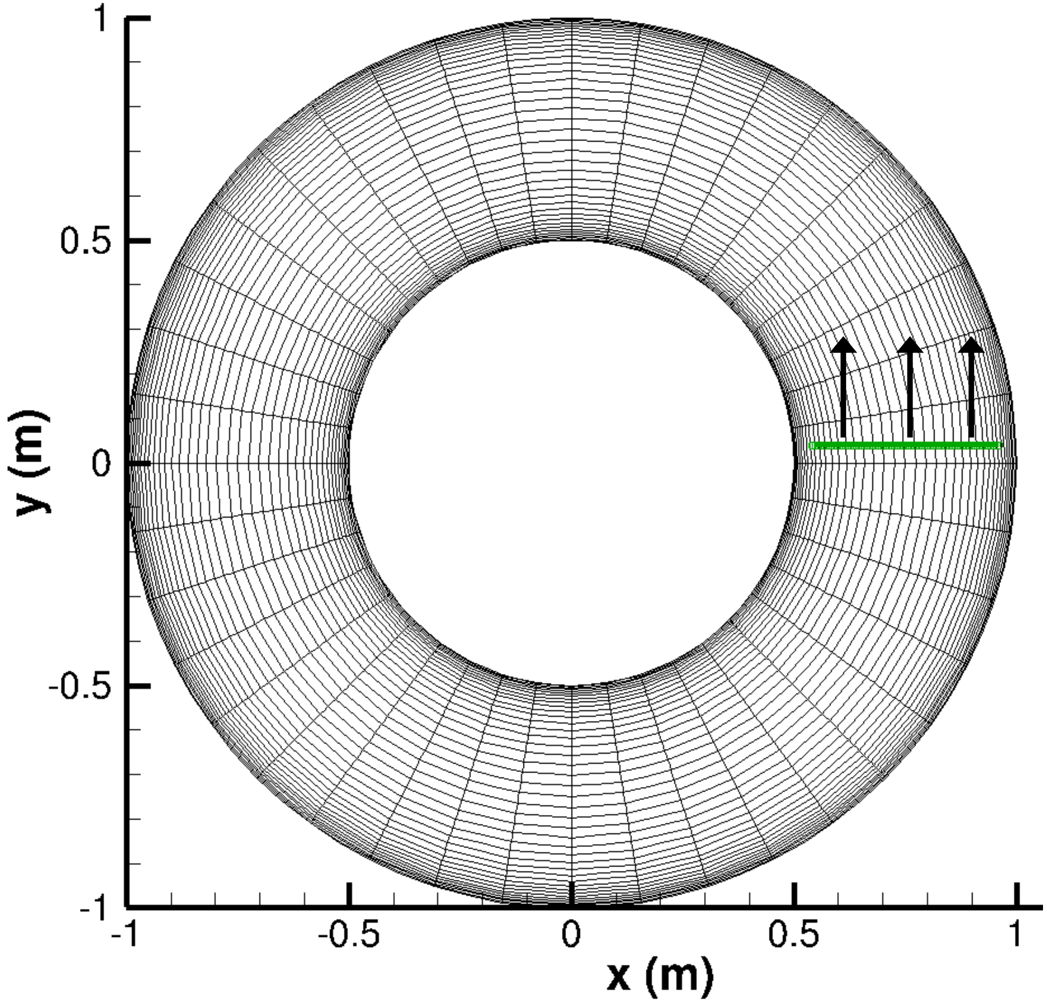




[1] Ching et al., *JCP*, 2020
[2] Ching and Ihme, *JCP*, submitted

Test case: particle advection in annular channel

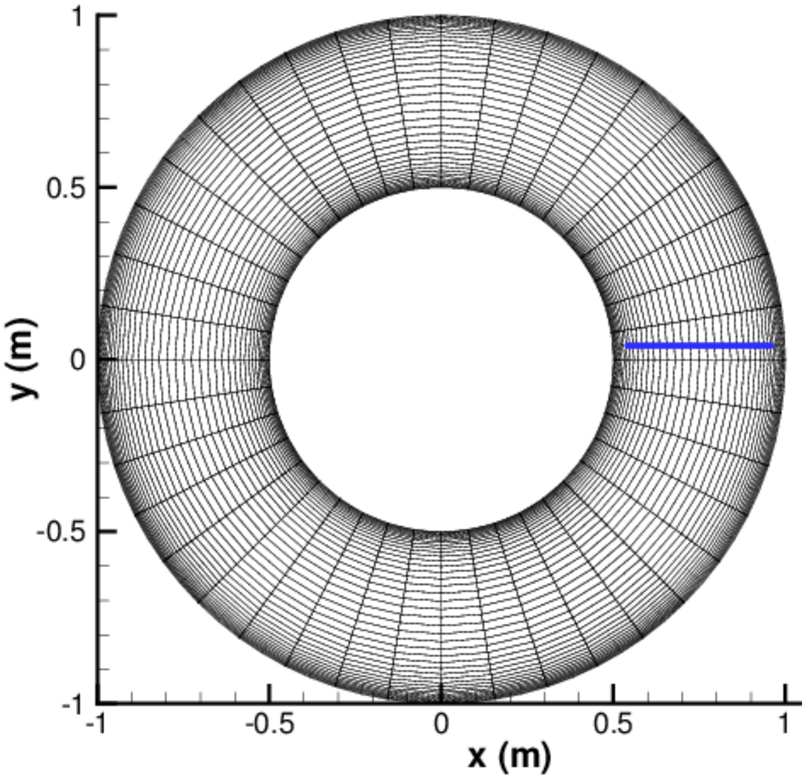
Compare straight-sided and curved elements

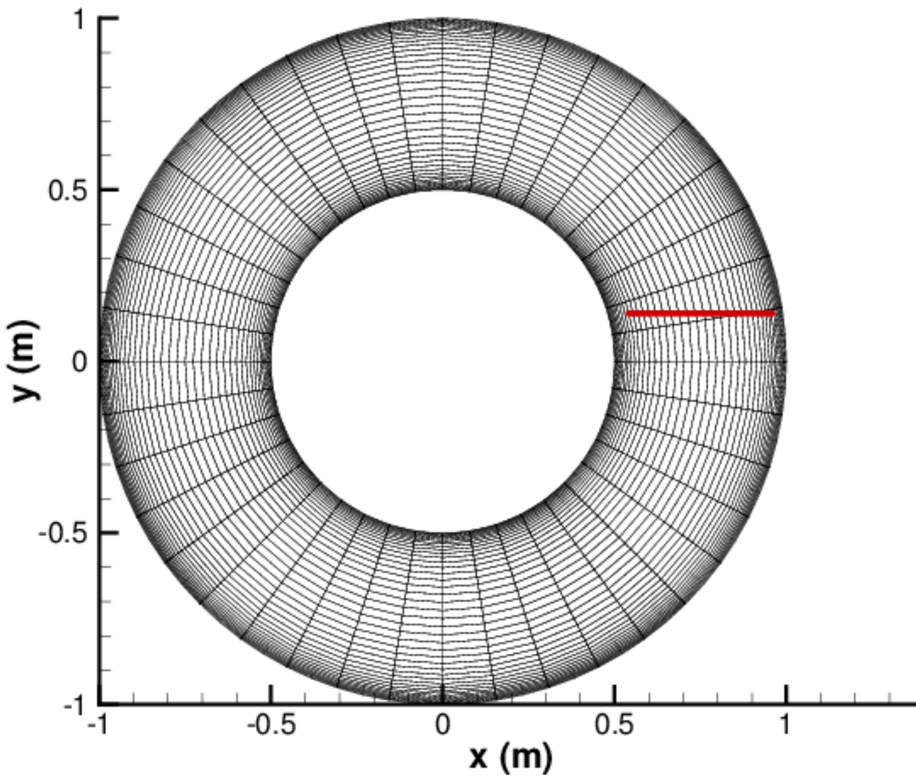


Test case: particle advection in annular channel

Blue: curved elements

Red: straight-sided elements





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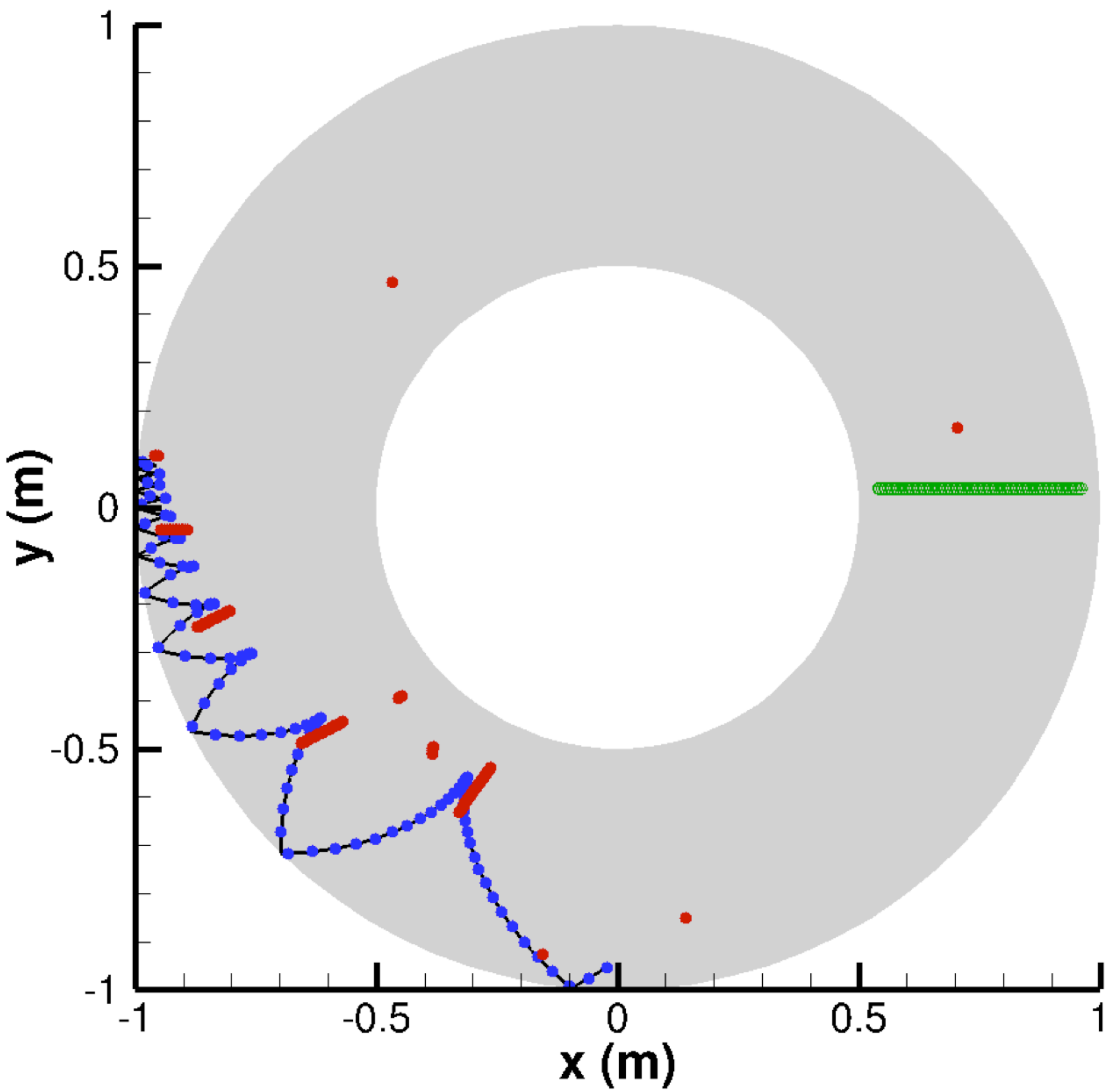


Goals

Develop an Euler-Lagrange DG methodology

Track and localize particles, including wall collisions (1-way)

Project influence of particles onto the Eulerian mesh (2-way)

Handle particle-particle collisions (4-way)

Efficient, accurate, and robust on **arbitrary curved, high-aspect-ratio** elements

Reverse coupling

Project the influence of the particles onto the Eulerian mesh

Source term $\boldsymbol{S} = [0, \boldsymbol{S}_m, S_e]^T$

$$\boldsymbol{S}_m = -\sum_{i=1}^{N_p} \boldsymbol{F}_i \chi(\boldsymbol{x}; \boldsymbol{x}_{d,i}),$$

$$S_E = -\sum_{i=1}^{N_p} \left(Q_i + \boldsymbol{u}_{d,i} \cdot \boldsymbol{F}_i \right) \chi(\boldsymbol{x}; \boldsymbol{x}_{d,i})$$

Projection kernel $\chi(\boldsymbol{x}; \boldsymbol{x}_{d,i})$

- $\int_\Omega \chi d\Omega = 1$
- $\arg\max_{\boldsymbol{x} \in \Omega} \chi(\boldsymbol{x}; \boldsymbol{x}_d) = \boldsymbol{x}_d$

Finite-volume context

- Box kernel[1]
- Volume-weighted[2], distance-weighted[3] kernel
- Isotropic Gaussian function[4]

[1] Crowe, *J Fluids Eng,* 1982
[2] Squires and Eaton, *Phys Fluid A*, 1990
[3] Elghobashi and Truesdell, *Phys Fluid A,* 1993
[4] Capecelatro and Desjardins, *J Comp Phys*, 2013

Reverse coupling

DG context

- Quadrature used to evaluate integrals
- Multiple DOFs per element, subcell resolution
- Modal basis functions
- More susceptible to numerical noise and instabilities
- Isotropic Gaussian function[1,2,3]

Shifted delta function[4] $\chi(\boldsymbol{x};\boldsymbol{x}_{d,i}) = \delta(\boldsymbol{x}-\boldsymbol{x}_{d,i})$

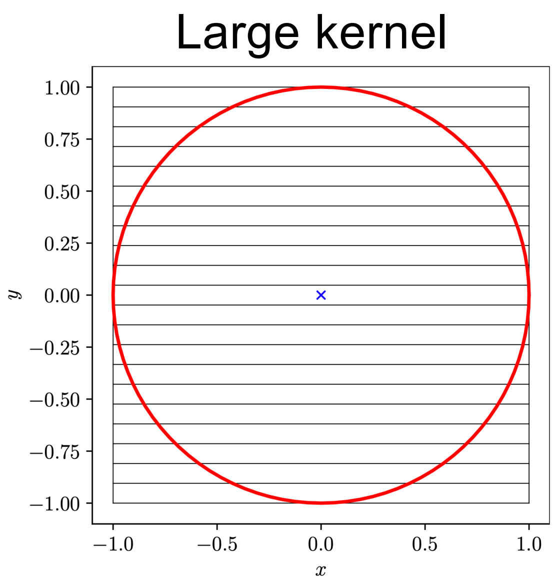
$$\int_{\Omega_e} \phi_m \boldsymbol{S} d\Omega_e = \sum_{i=1}^{N_p^e} \phi_m(\boldsymbol{x}_{d,i})[0, \boldsymbol{F}_i, Q_i + \boldsymbol{u}_{d,i}\cdot\boldsymbol{F}_i]^T$$

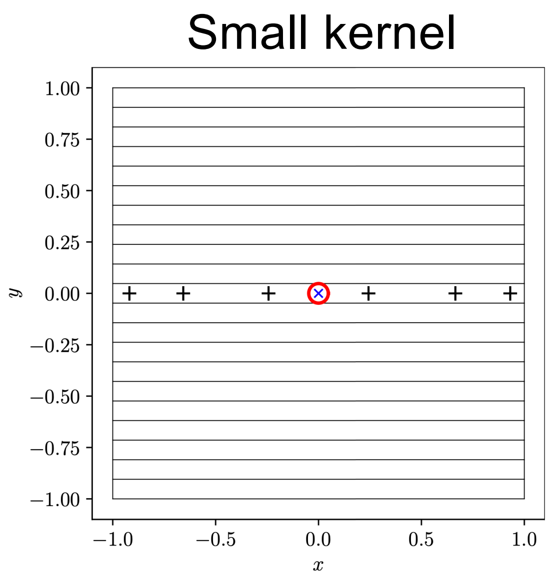
- Simple
- Not smooth → numerical noise
- Inappropriate for larger particles

[1] Jacobs and Hesthaven, *J Comp Phys*, 2006
[2] Huang et al, *Comput Methods Appl Mech Eng*, 2019
[3] Zwick and Balachandar, *Int J High Perform Comput Appl*, 2020
[4] Ching et al., *J Comp Phys*, 2020

Smooth isotropic kernels: challenges

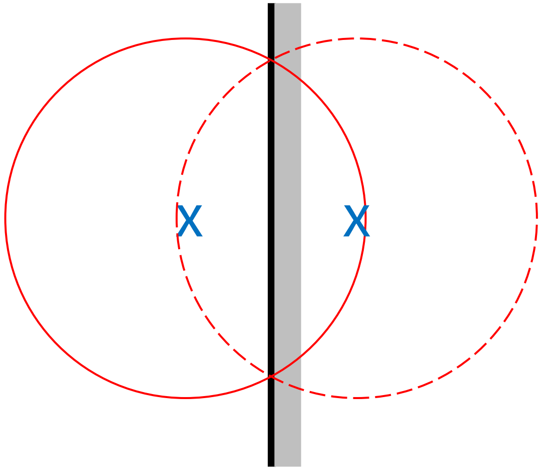
High-aspect-ratio elements





Curved elements

- To conserve **near-wall interphase transfer**, common to use mirror particles on straight-sided elements
- Not straightforward for arbitrary curved elements

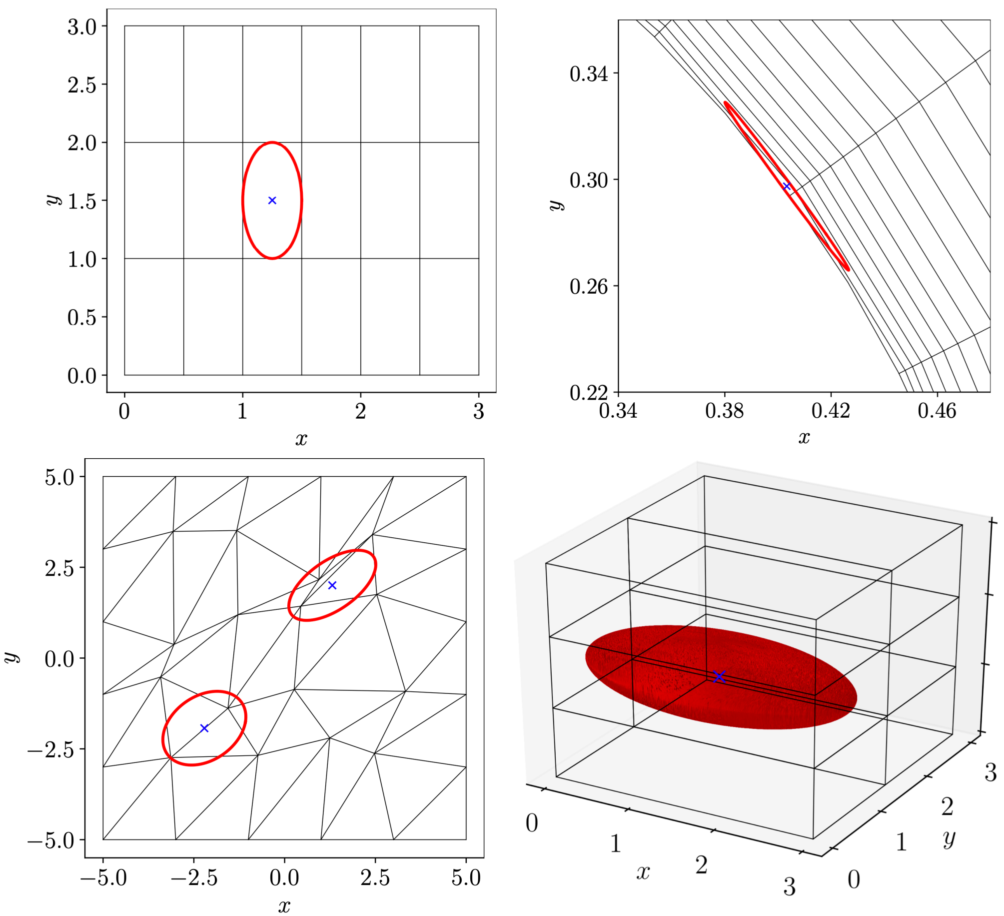


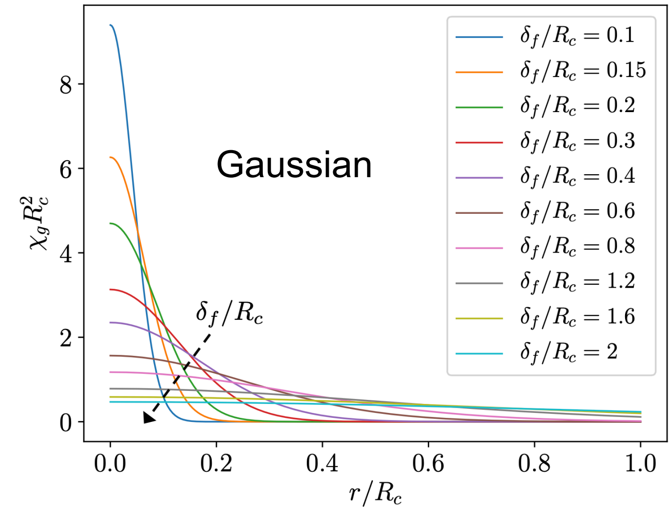
Proposed methodology

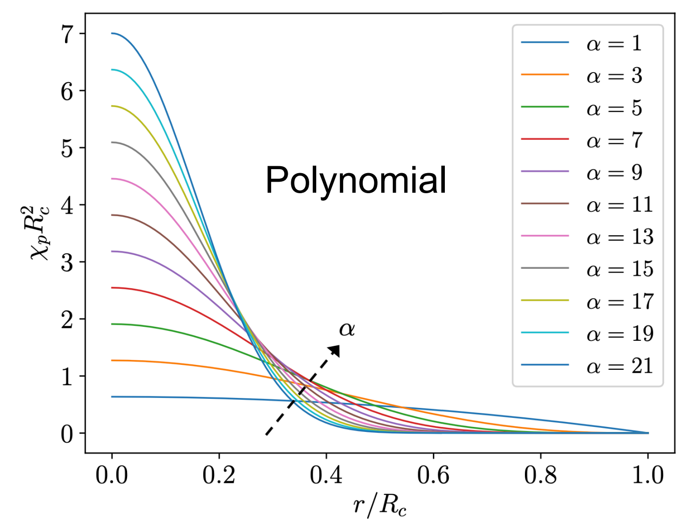
1. Smooth anisotropic kernels
   - Balance:
     - Accuracy
     - Mitigation of numerical noise and instabilities
     - Computational cost
   - Elliptical/ellipsoidal (instead of circular/spherical) in 2D/3D
2. Efficient kernel rescaling near walls
   - Avoid brute-force approach
   - Employ high-order polynomials to approximate rescaling factor

Smooth anisotropic kernel

- Construct ellipse based on mesh-implied metric[1,2,3]
- Polynomial-based kernel[4] instead of Gaussian-type kernel







[1] Fidkowski, MIT thesis, 2007
[2] Oliver, MIT thesis, 2008
[3] Sanjaya et al., AIAA Paper, 2020
[4] Jacobs and Hesthaven, *JCP*, 2006

Near-wall treatment

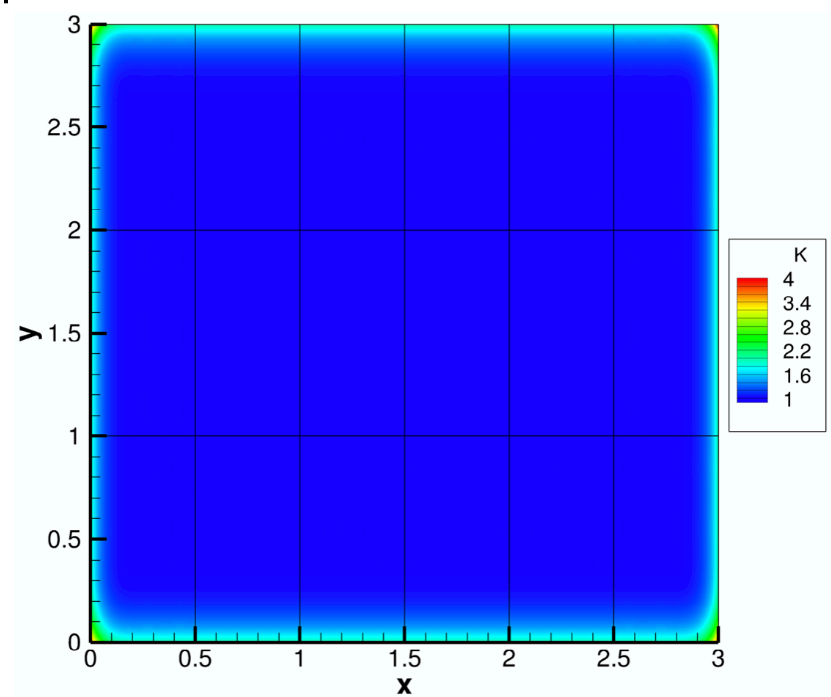
Brute-force approach: exact rescaling

$$K(\boldsymbol{x}_d) = \frac{1}{\sum_{e=1}^{N_v} \int_{\Omega_e} \chi_p(\boldsymbol{x}; \boldsymbol{x}_d) d\Omega}$$

Proposed approach: approximate rescaling

- Construct high-order polynomial approximation of *K*
  - Sample *K* at quadrature points
  - Project to high-order polynomials

$$K_h^e(\boldsymbol{x}_d) = \sum_{n=1}^{N_K} \widetilde{K}_n^e \Phi_n(\boldsymbol{x}_d)$$

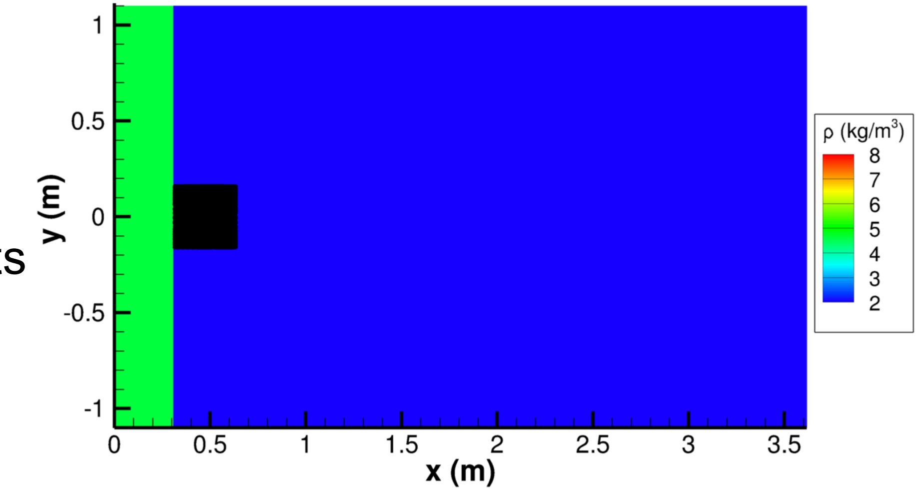


[1] Ching and Ihme, *JCP*, submitted

Test case: shock interaction with particle cloud

- Bronze particle cloud
- Volume fraction = 4%

- 500,000 2D quadrilateral elements
- *p* = 3 polynomials

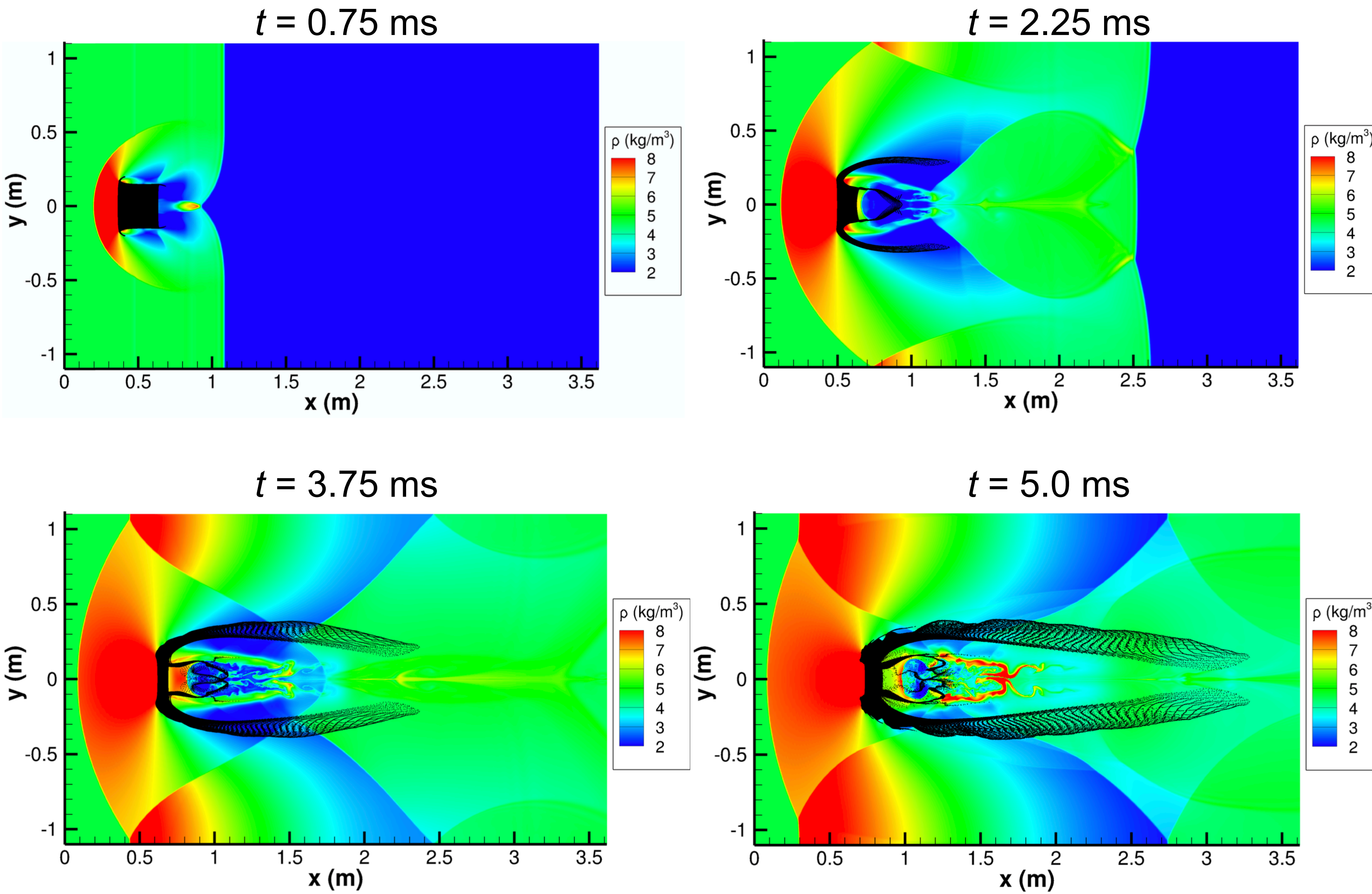


ρ_d (kg/m^3)	D (µm)	c_d (J/kg/K)	ρ_c (kg/m^3)	u_c (m/s)	P_c (bar)
8900	100	435	1.2	0	1

- Difference between delta shape function and smooth anisotropic kernel
- Good agreement with numerical results[1]

[1] Kiselev et al., *Shock Waves*, 2006

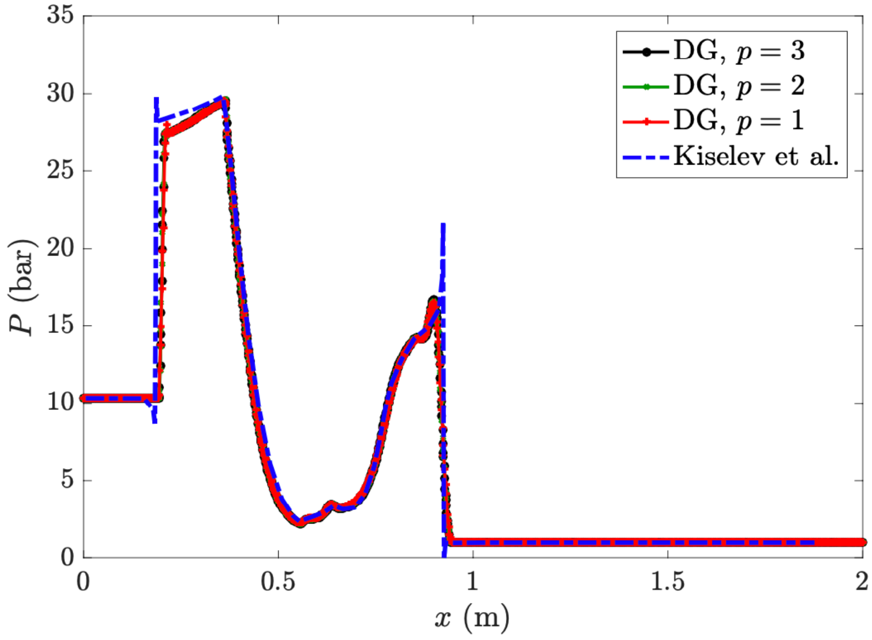
Test case: shock interaction with particle cloud



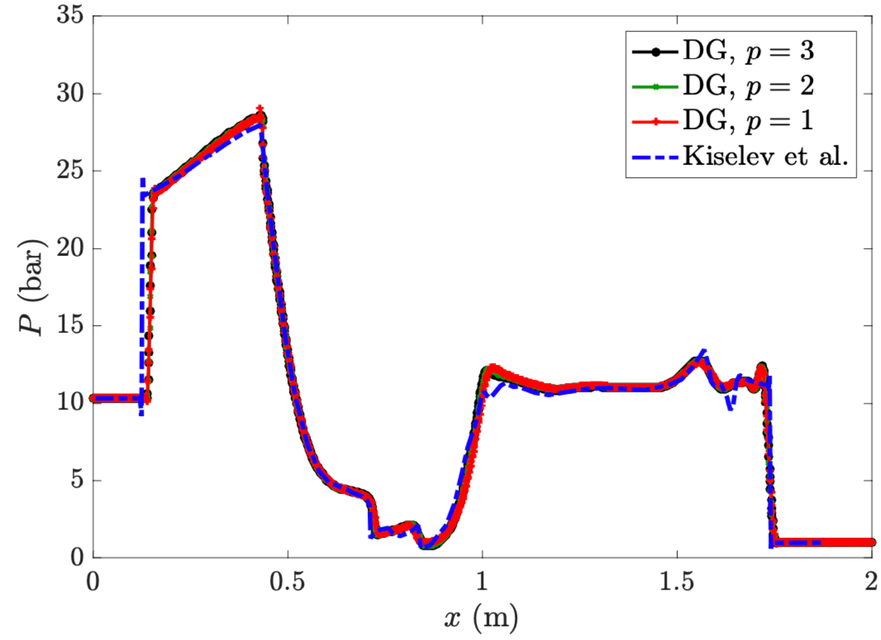
Test case: shock interaction with particle cloud

Comparison with numerical results

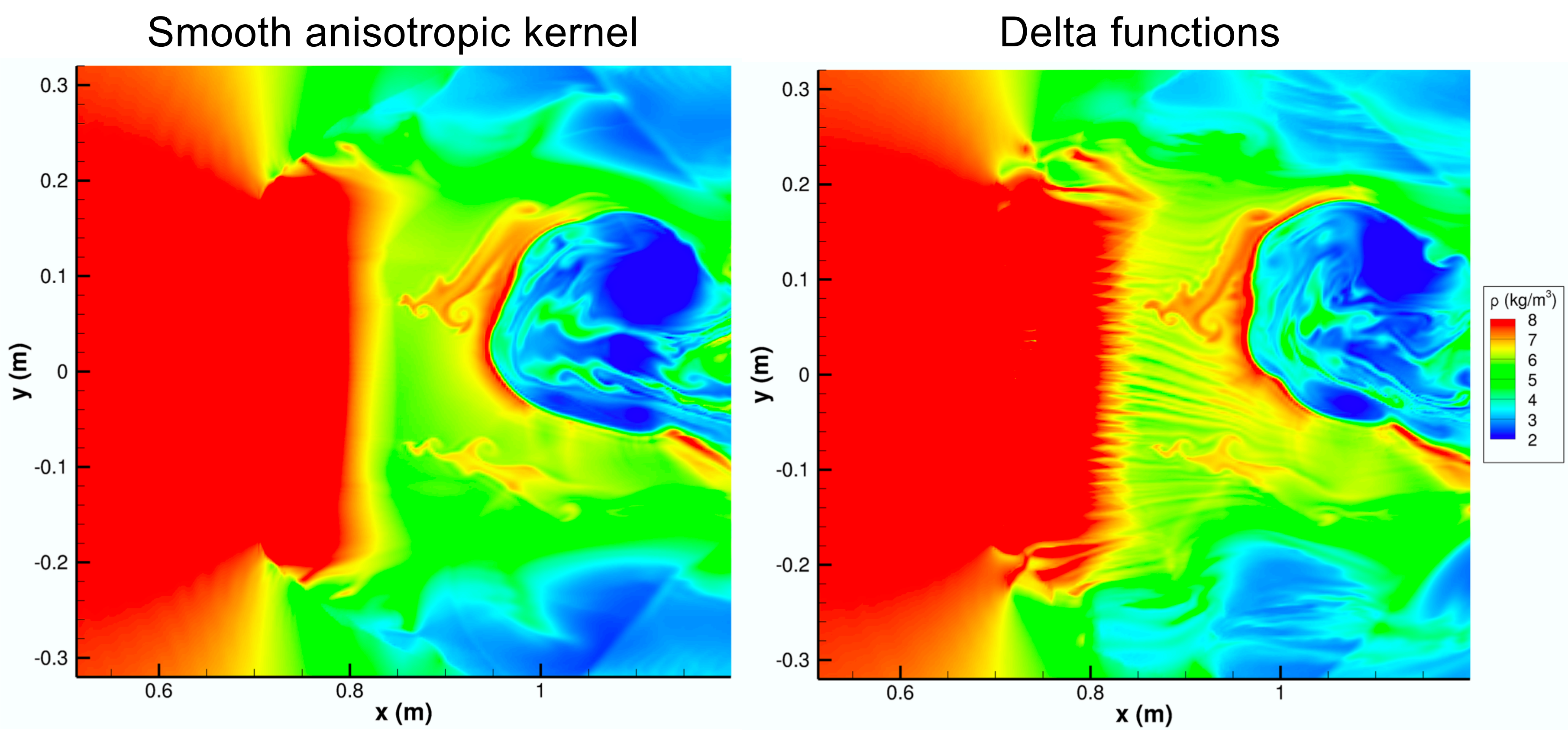
t = 0.75 ms



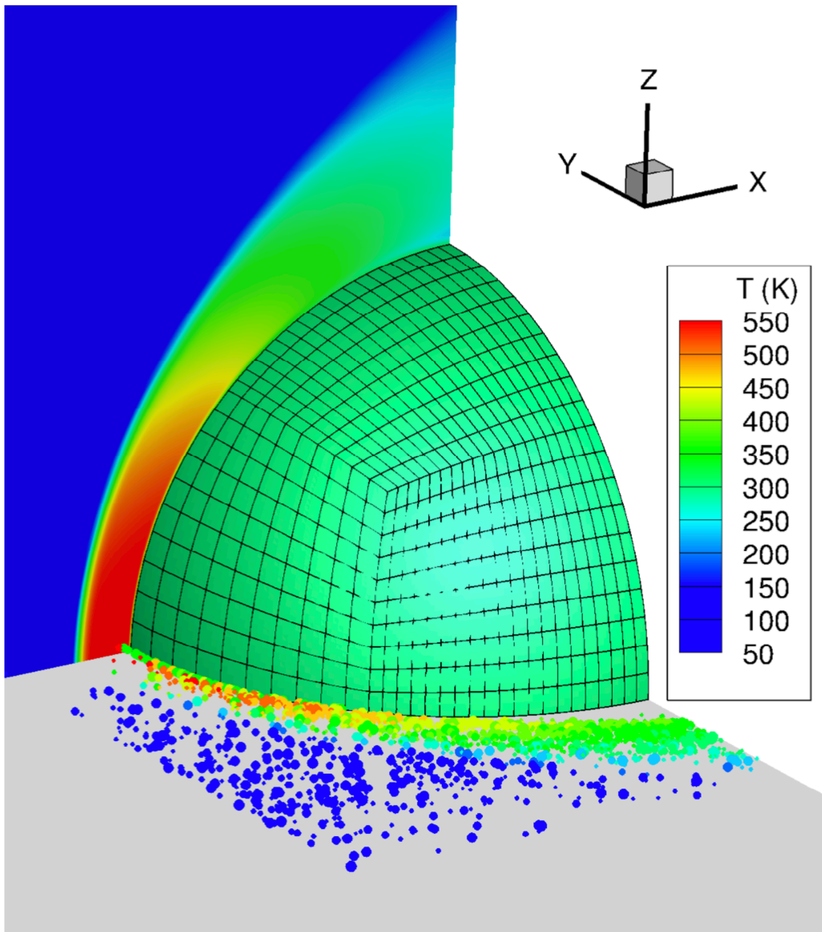
t = 1.5 ms

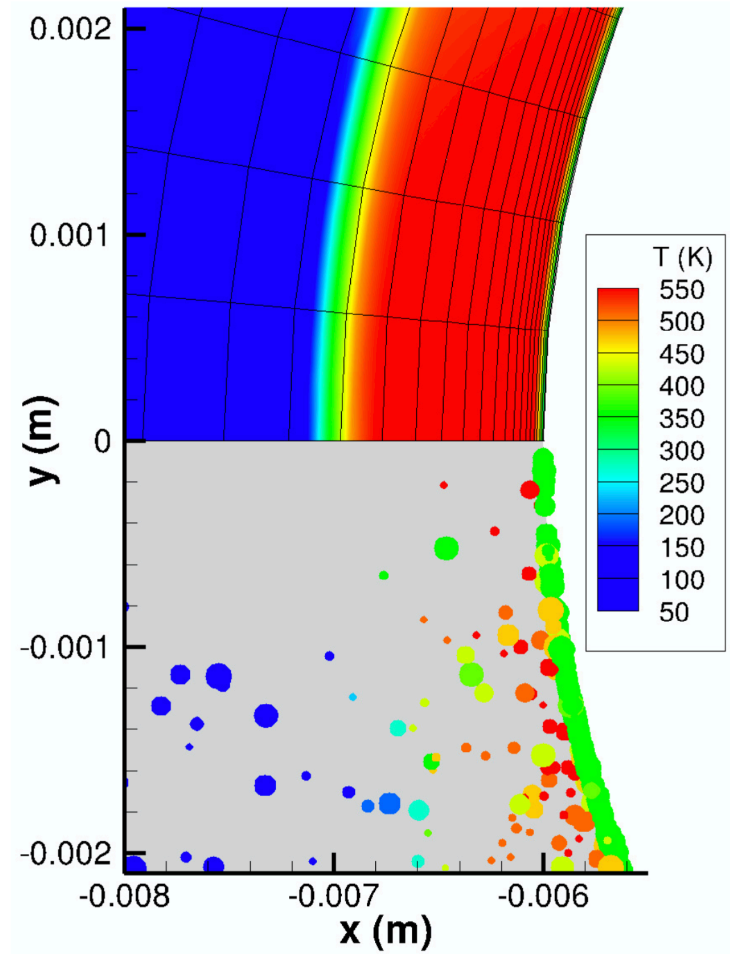


Test case: shock interaction with particle cloud

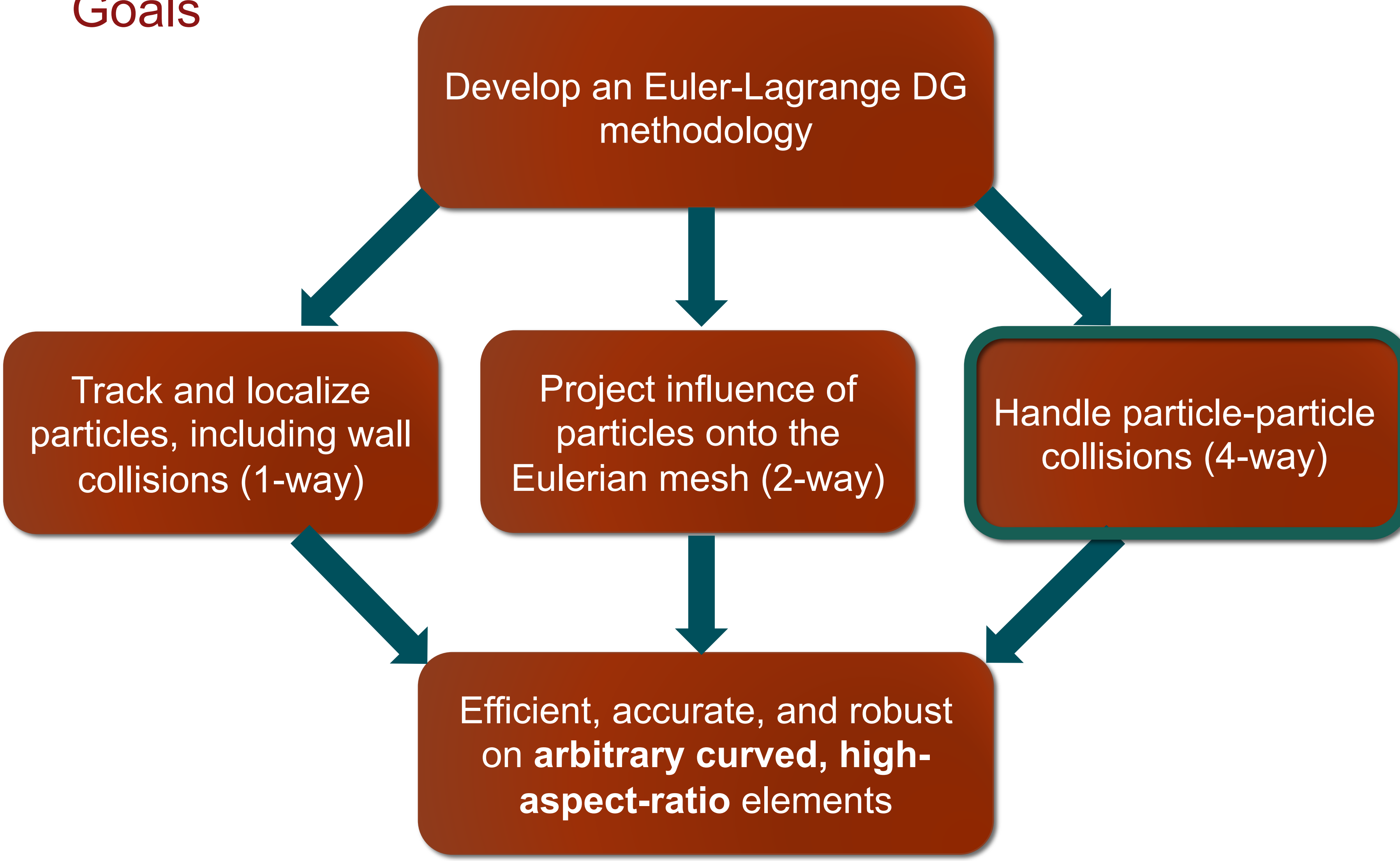


Hypersonic dusty flows over blunt bodies



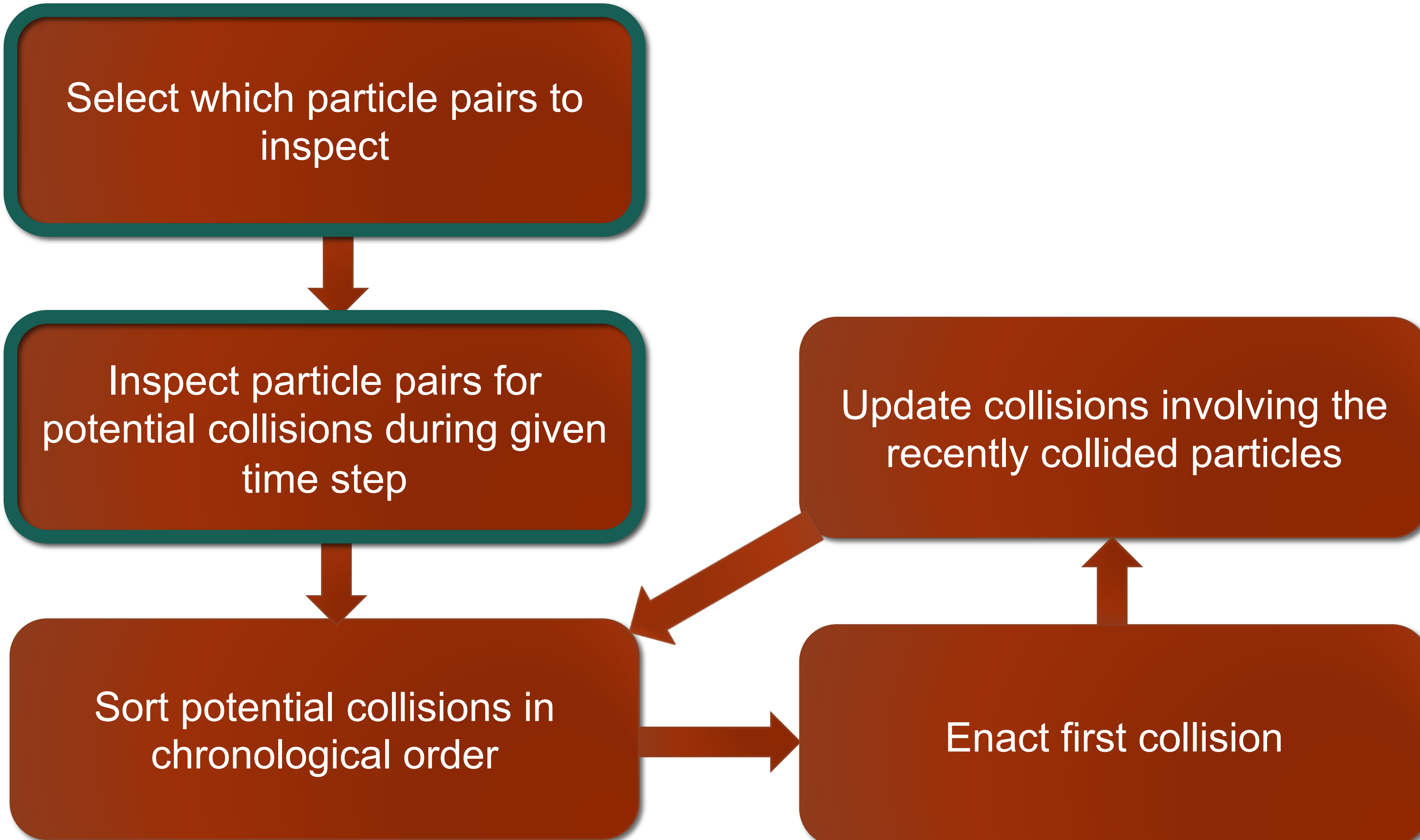


Goals


Develop an Euler-Lagrange DG methodology
Track and localize particles, including wall collisions (1-way)
Project influence of particles onto the Eulerian mesh (2-way)
Handle particle-particle collisions (4-way)
Efficient, accurate, and robust on **arbitrary curved, high-aspect-ratio** elements

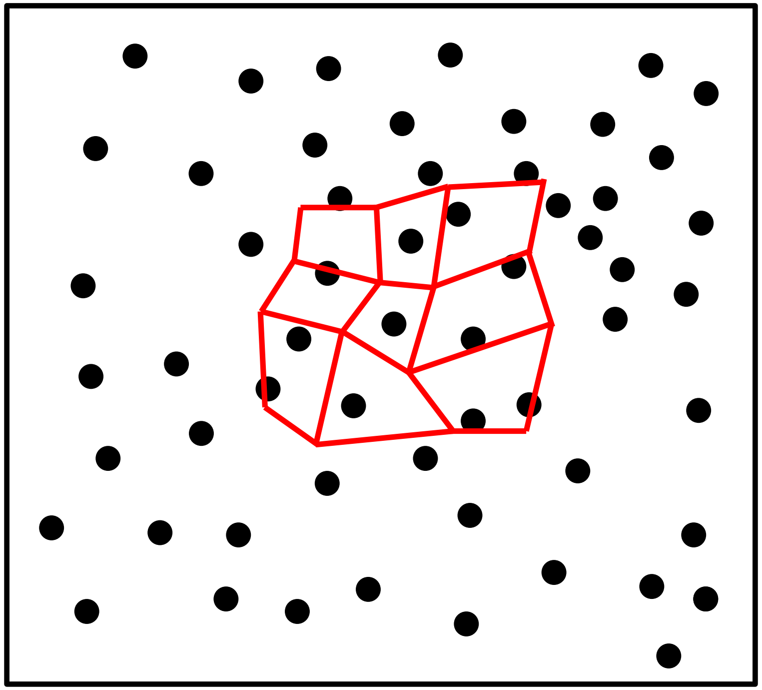
Particle-particle collisions

Hard-sphere model



Selecting which particle pairs to inspect

- Brute-force approach
  - All possible particle pairs in domain
- Element neighbor lists
  - Inspect particles in node-sharing elements

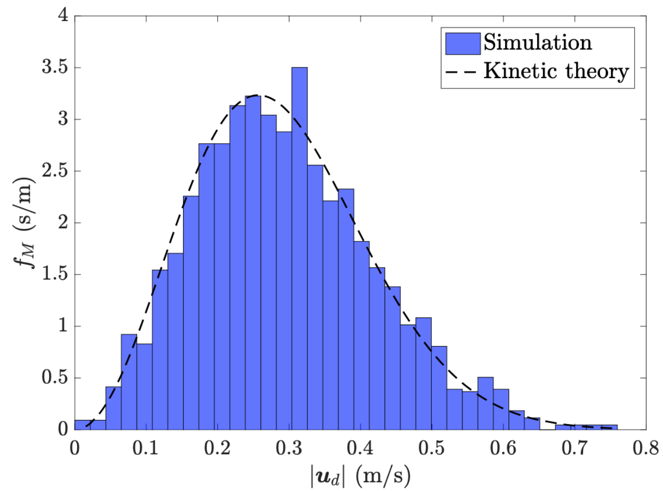


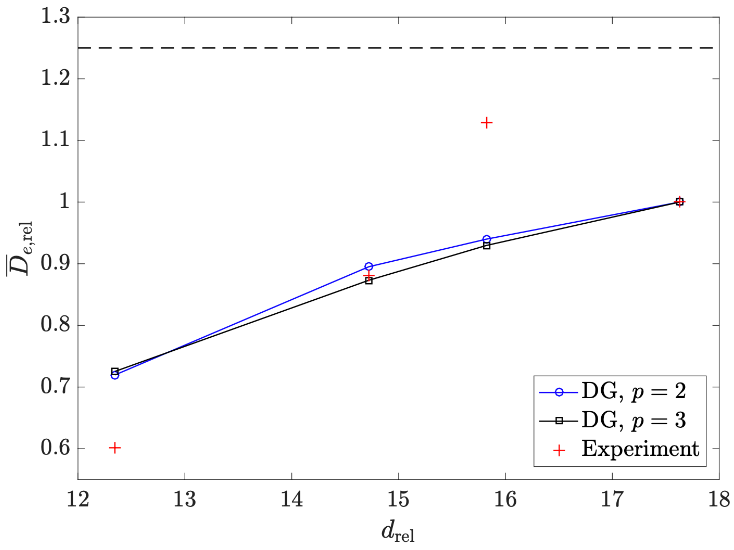
Truncate element neighbor lists

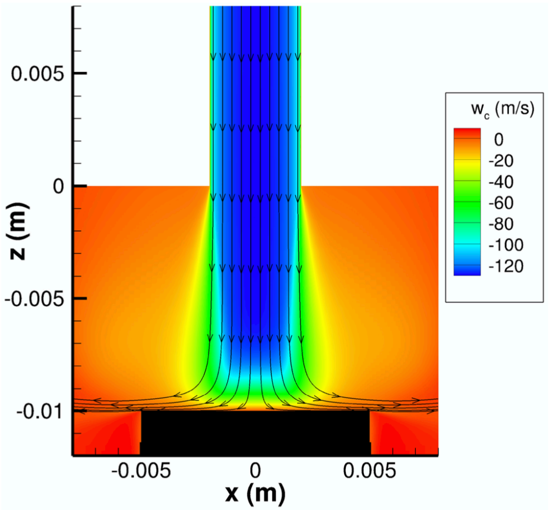
- Exploit the geometric mapping to further localize pairings
- Compatible with unstructured curved elements
- Simple and fast truncation process

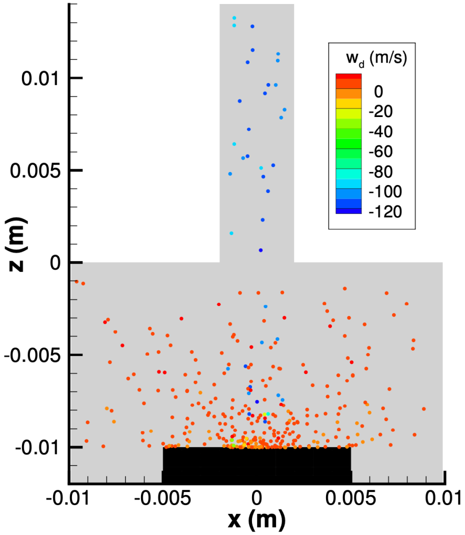
Test cases

- 2D/3D kinetic theory example
  - Recover equilibrium properties
  - Significant speedup
- 3D sandblasting example
  - Impingement of particle stream on a flat plate
  - Effect of particle mass flux on erosion
  - Good agreement with experiments[1]









[1] Oka et al., Wear, 2001

Summary

Developed an Euler-Lagrange DG solver

- Simple and robust shock capturing
  - Accurate surface heating predictions in hypersonic flows
- Lagrange particle methods for 1-way, 2-way, 4-way coupling
  - Compatible with arbitrary curved, high-aspect-ratio elements

Why DG?

- DG can considerably simplify meshing of complex geometries
- Curved elements can significantly improve predictions of particle trajectories

Outline

Background

- Introduction & Motivation
- DG mathematical background

Development

- Shock capturing scheme
- Lagrangian particle method

Application

Hypersonic dusty flows over blunt bodies

- Experimental data and parametric sensitivities
- Mars atmospheric entry

Hypersonic dusty flows over blunt bodies

Relevant to Mars atmospheric entry

Adverse effects[1]:

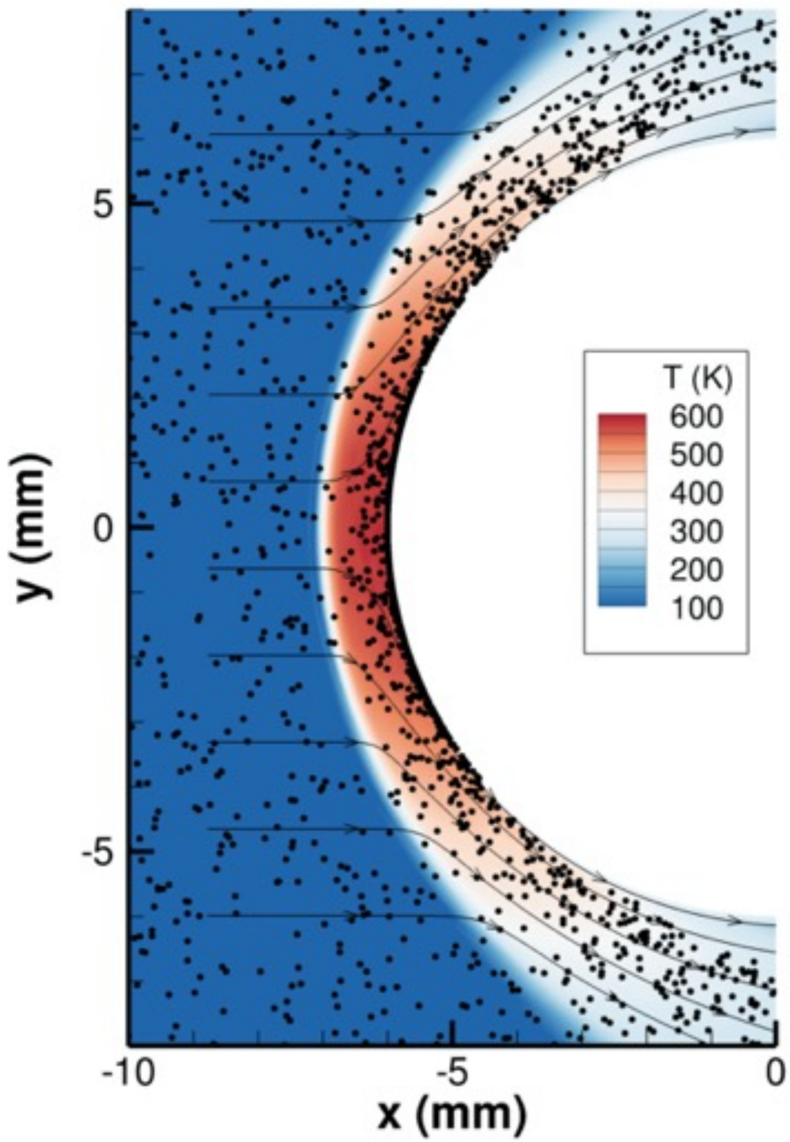
- Surface erosion
  - Particle-wall collisions
- Surface heat flux augmentation
  - Particle-wall collisions
  - Two-way coupling

Limited high-quality experimental data

- Ambiguity in how to reliably model these flows

Goals:

- Simulate the hypersonic dusty flow experiment by Vasilevskii et al.[2]
- Evaluate solution sensitivities to physical modeling of the particle phase



[1] Montois et al., International Planetary Probe Workshop, 2007
[2] Vasilevskii et al., *J Eng Phys Thermophys*, 2001.

Experiments by Vasilevskii et al.

Hypersonic dusty flow over a sphere

Gas	Ma_∞	$P_{t,\infty}$ (bar)	$T_{t,\infty}$ (K)	R_s (m)
N_2	6.1	17.5	570	0.006

Dust material	ρ_d (kg/m^3)	$\overline{D}$ (m)	β
SiO_2	2264	0.19×10^{-6}	0.03

Measured ratio of dusty-gas to pure-gas heat flux at stagnation point

Solver details

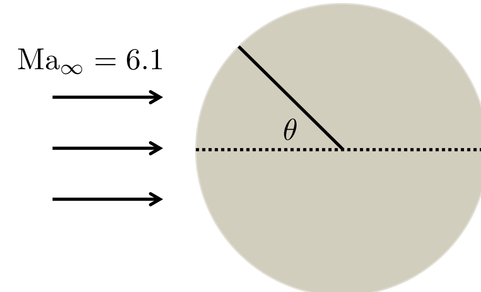
- p = 2
- 60,000 elements
- Implicit-explicit time stepping
  - BDF3-AB3
- Two-way coupling

[1] Vasilevskii et al., *J Eng Phys Thermophys*, 2001.

Baseline physical model

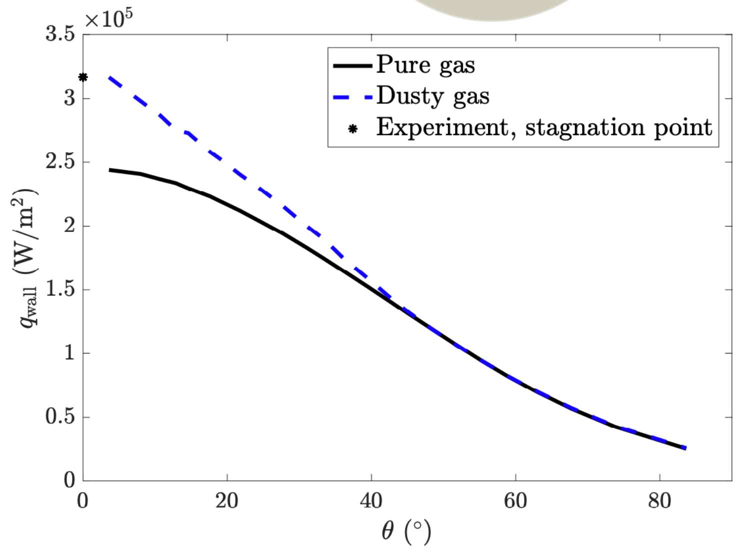
Simplest model that gives good agreement with experiment

- Henderson drag correlation[1]
- Fox Nusselt number correlation[2]
- Thermophoretic force[3]



$$m_d \frac{d\boldsymbol{u}_d}{dt} = \boldsymbol{F} = \boldsymbol{F}_{qs} + \boldsymbol{F}_{th}$$

$$m_d c_d \frac{dT_d}{dt} = Q = Q_{qs}$$

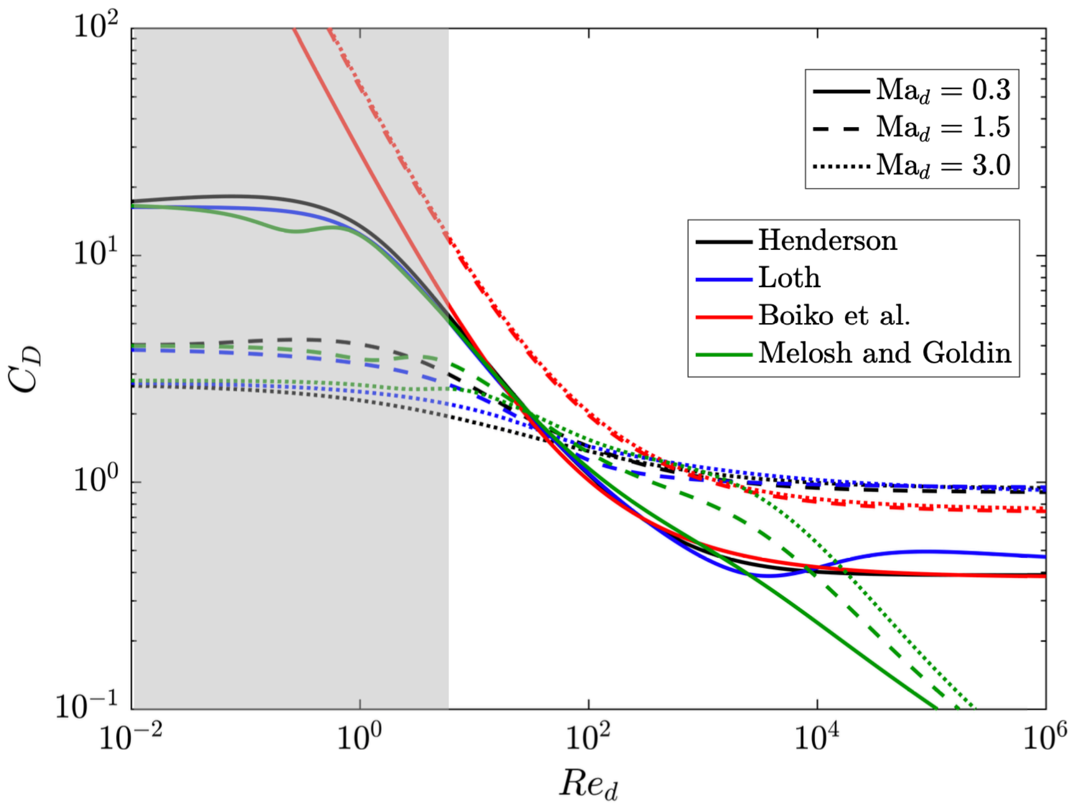


Sensitivity study to be conducted with respect to this baseline model

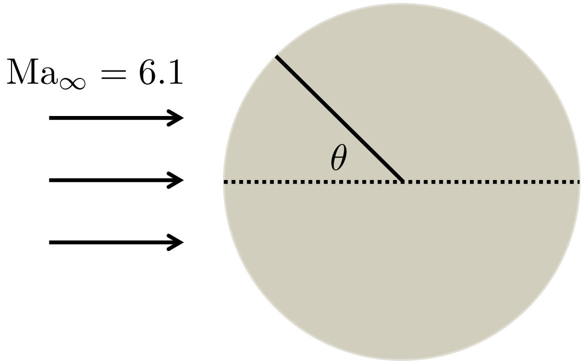
[1] Henderson, *AIAA J*, 1976
[2] Fox et al., 11th International Shock Tubes and Waves Symposium, 1978
[3] Loth, *AIAA J*, 2008

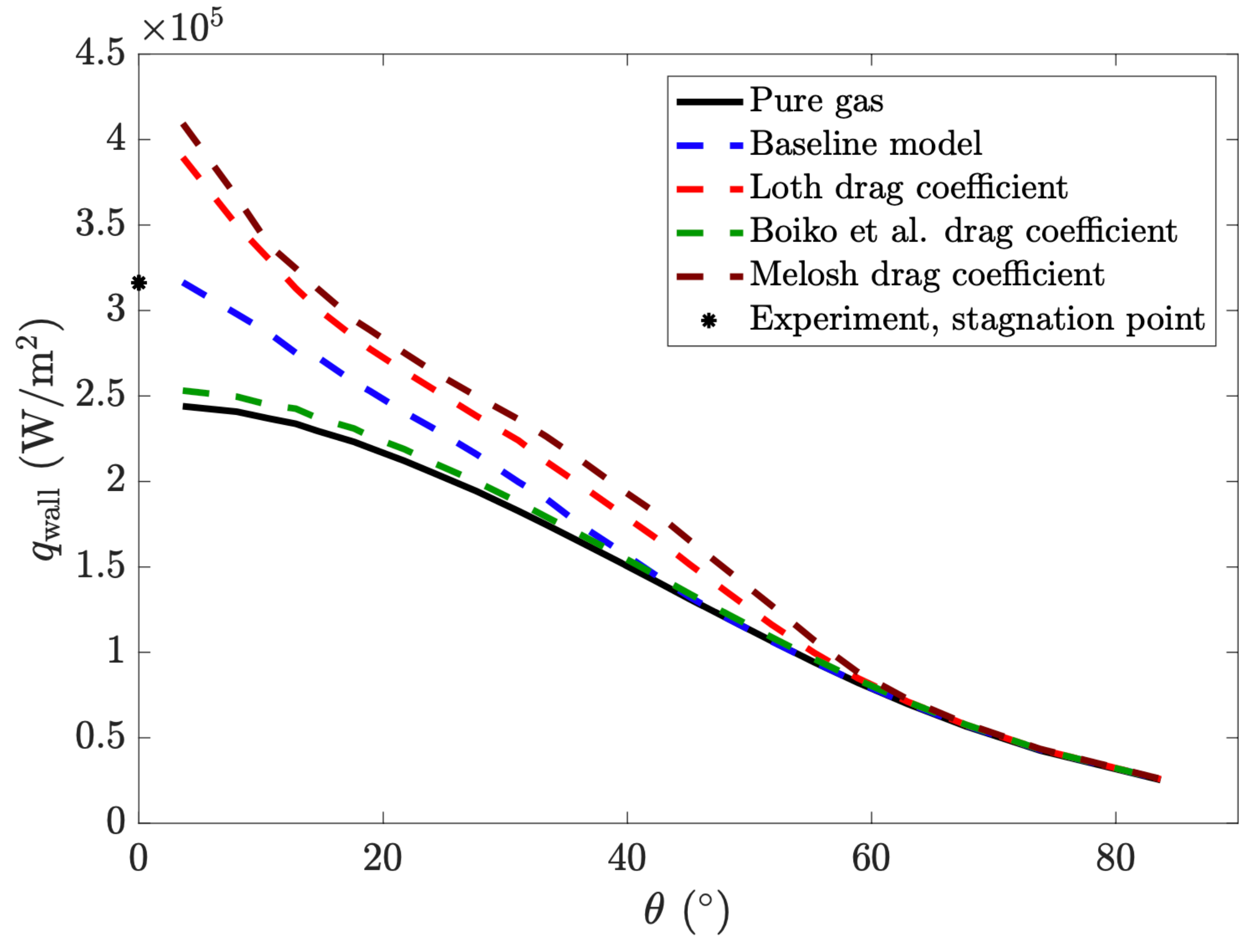
Drag correlations

- Henderson (*AIAA J*, 1976)
- Loth (*AIAA J*, 2008)
- Boiko et al. (*Shock Waves*, 1997)
- Melosh and Goldin (Lunar and Planetary Science, 2008)



Sensitivity to drag correlation

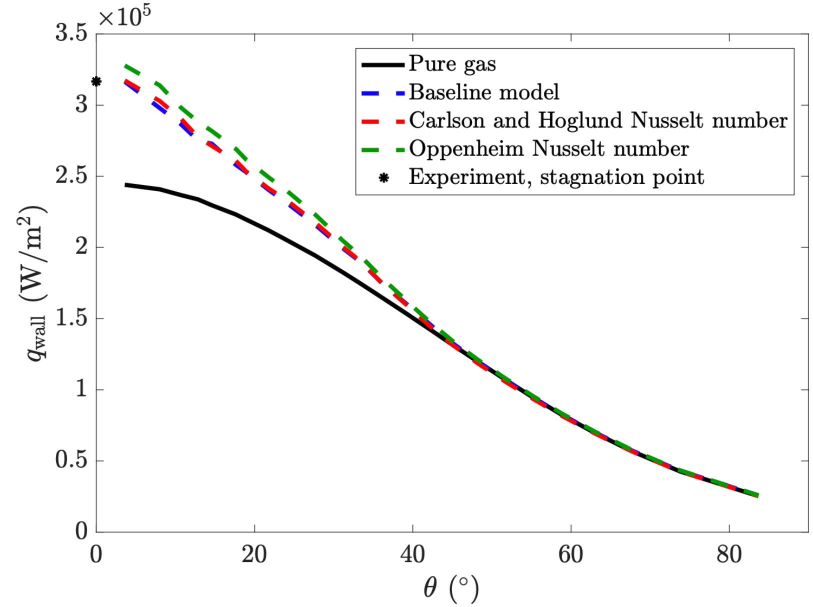


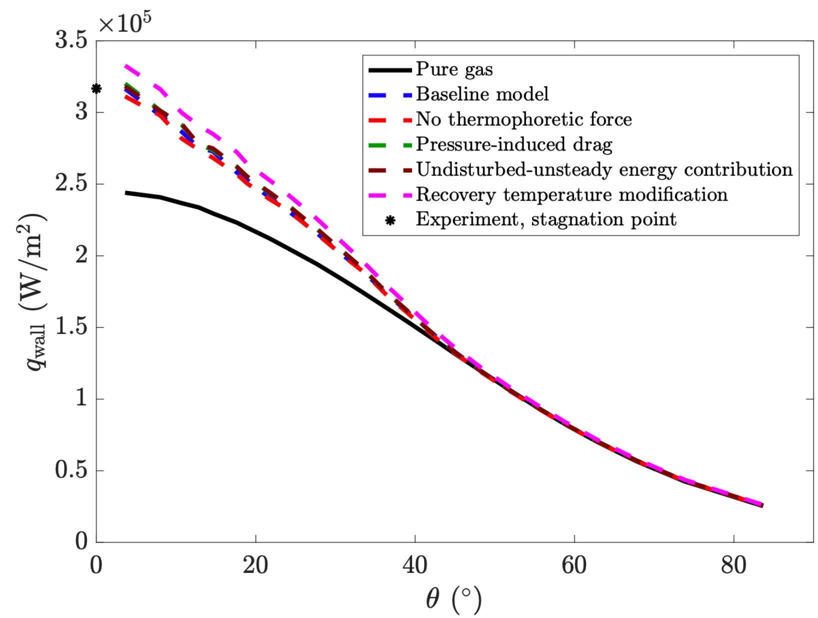


[1] Ching et al., *J Spacecraft Rockets*, accepted

Additional sensitivities

- Nusselt number correlations
  - Small sensitivity
- Momentum and energy contributions
  - Quasi-steady drag and heating most important

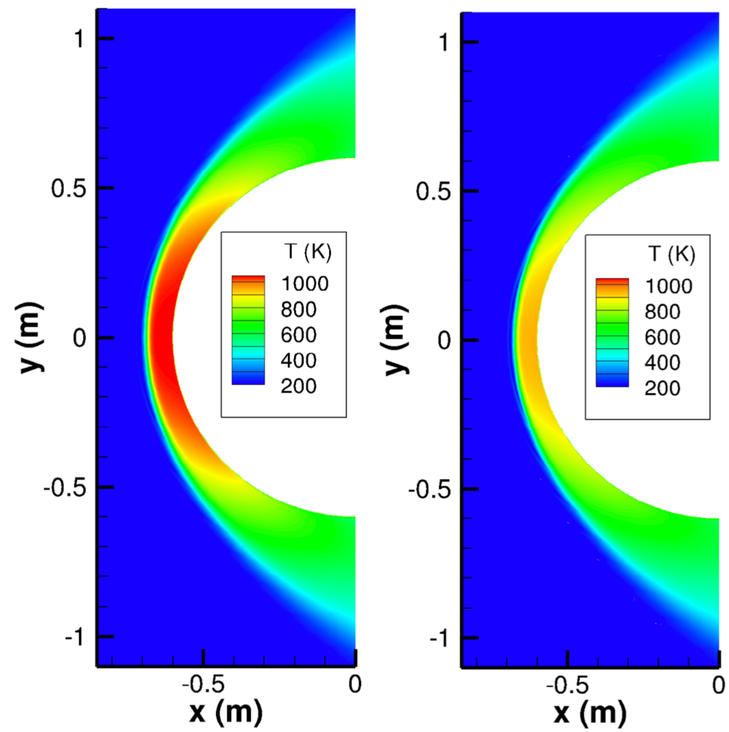




[1] Ching et al., *J Spacecraft Rockets*, accepted

Additional sensitivities

- Nusselt number correlations
  - Small sensitivity
- Momentum and energy contributions
  - Quasi-steady drag and heating most important
- Gas model
  - Shock standoff distance and shock layer quantities can have significant influence

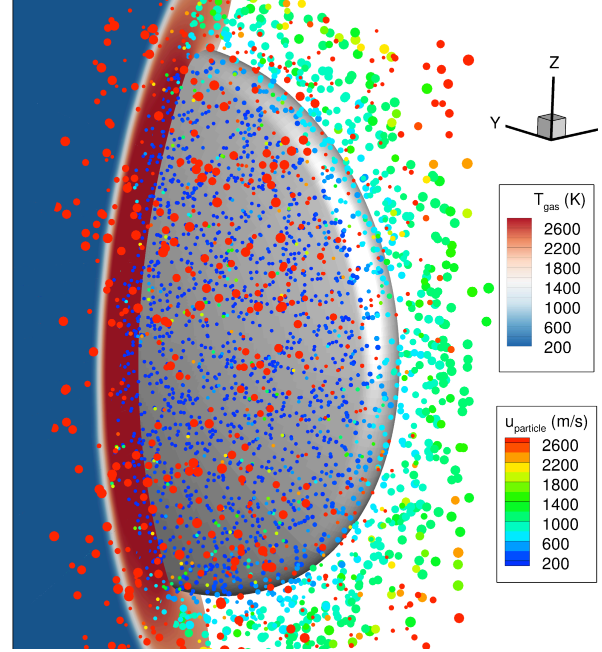


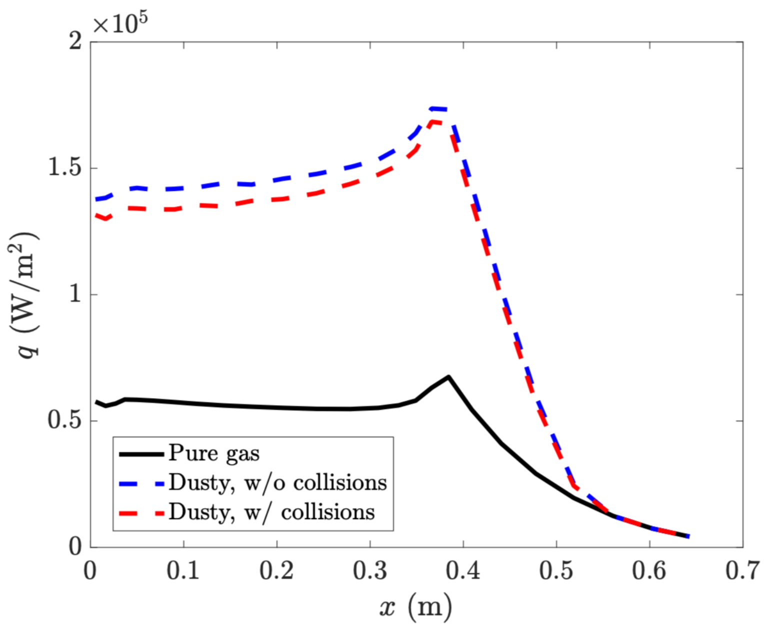
[1] Ching et al., *J Spacecraft Rockets*, accepted
[2] Ching et al., *Comput Fluids*, submitted

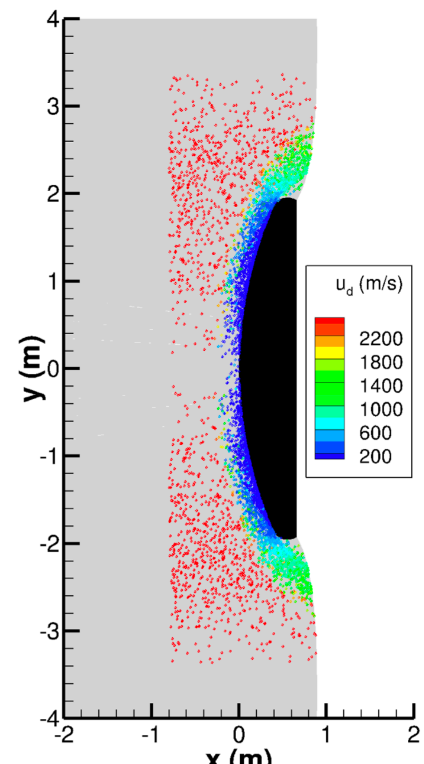
Additional sensitivities

Particle-particle collisions

- Hypersonic dusty flow over a capsule forebody
- Can attenuate particle-wall collisional energy flux
- Only important at very high dust loading







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- Experimental data and parametric sensitivities
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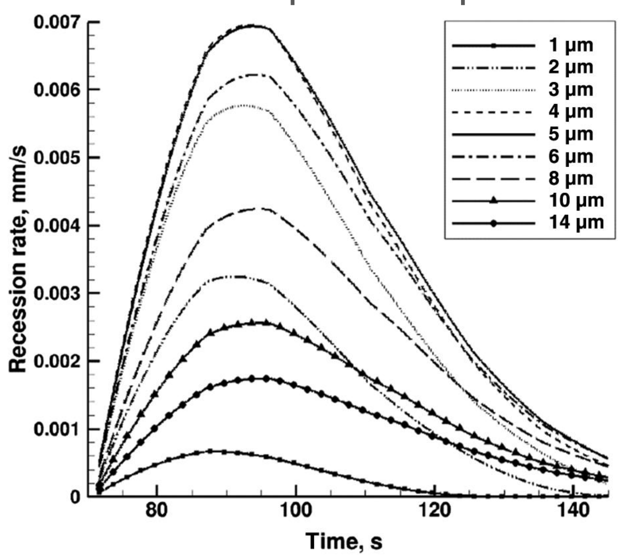
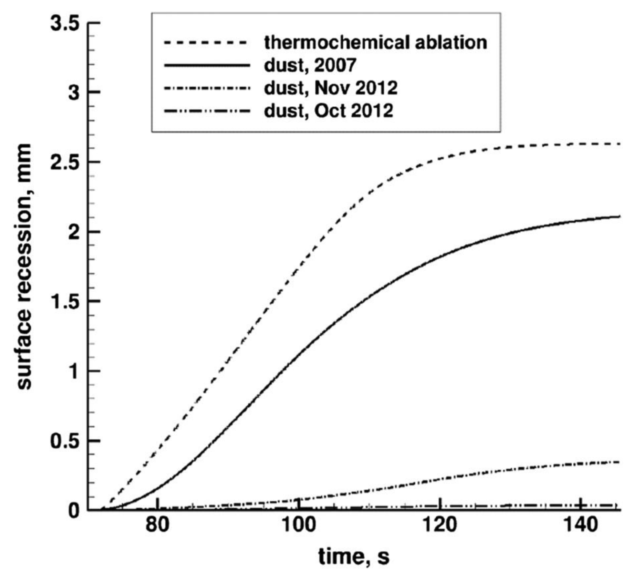
Background

Build on:

- Previous section
- Palmer et al., *Journal of Spacecraft and Rockets*, 2020
  - Computed heat shield recession at the stagnation point during the trajectory of the ExoMars Schiaparelli capsule





Goals:

- Further improve understanding of relevant physical processes
- Assess effects of particle size distribution, angle of attack, and two-way coupling on heating augmentation and erosion
- Employ recently developed physics-based drag correlation[1]
- Investigate particle trajectories in aft region

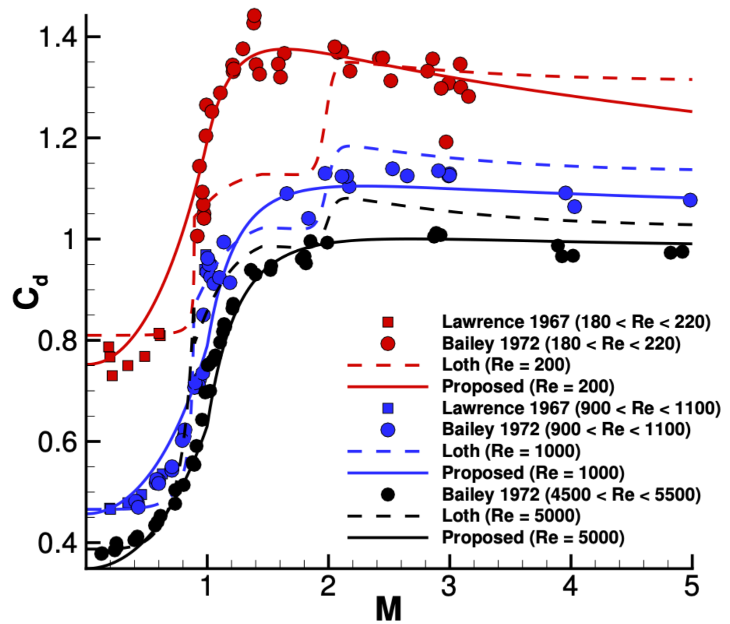
[1] Singh et al., https://arxiv.org/abs/2012.04813, 2020

Drag correlation

Recently developed physics-based drag correlation[1] that incorporates:

- Low-speed hydrodynamics
- Shockwave physics
- Rarefied gas dynamics

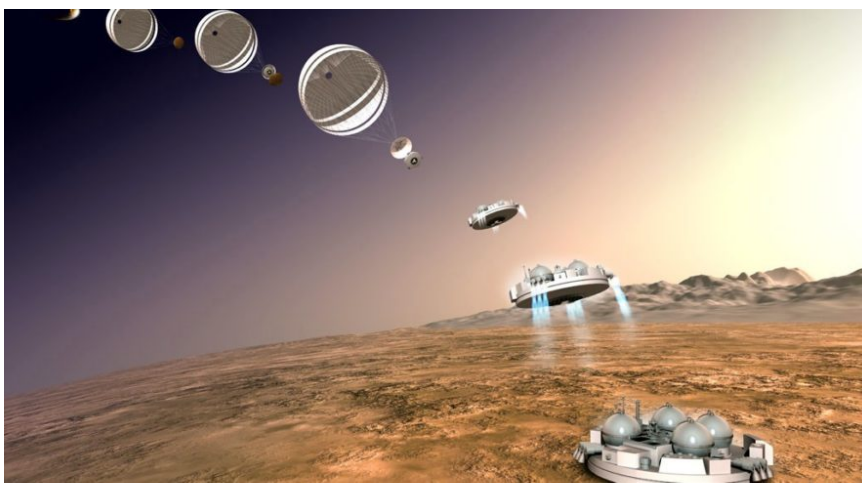
Better agreement with experimental data than popular correlations[2,3]

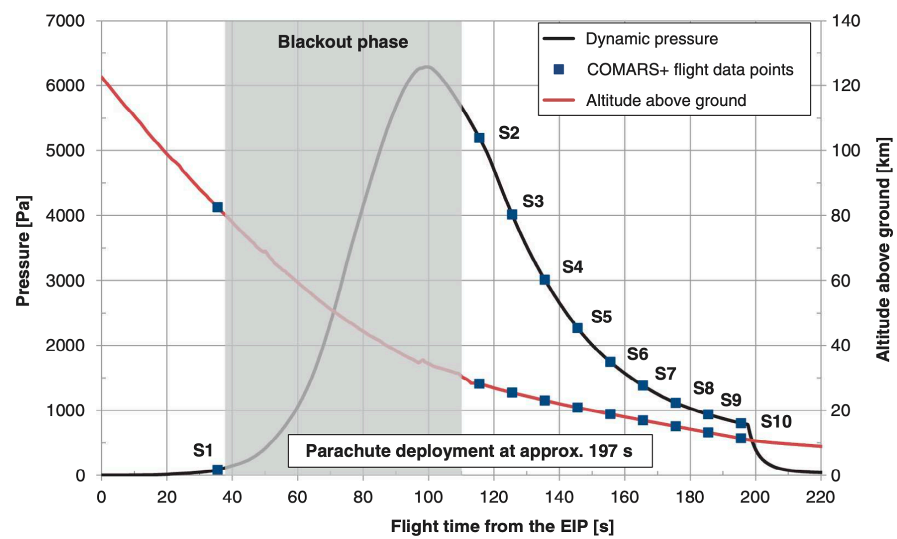


[1] Singh et al., https://arxiv.org/abs/2012.04813, 2020
[2] Henderson, AIAA J, 1976
[3] Loth, AIAA J, 2008

ExoMars Schiaparelli trajectory

- 1st mission of the European Space Agency's ExoMars program
- **Goals**:
  - Search for evidence of methane and other gases that could signify biological or geological processes
  - Test key technologies to support future missions





ExoMars Schiaparelli trajectory

S3 trajectory point[1]

Gas	Mach	u (m/s)	P (Pa)	T (K)
CO_2	8.97	2014	74.1	195

Two angles of attack: 0 and 20 deg

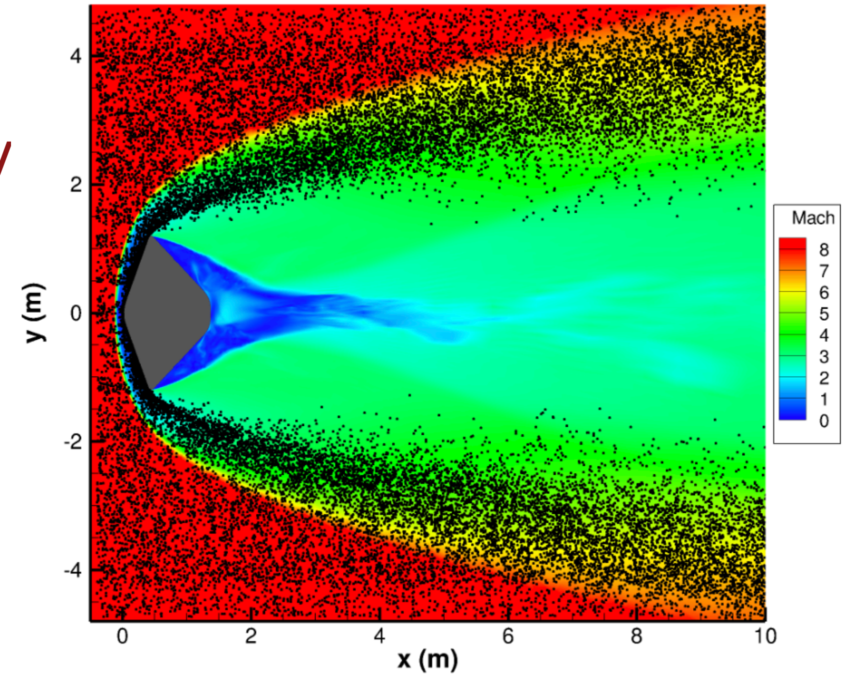
SiO_2 particles

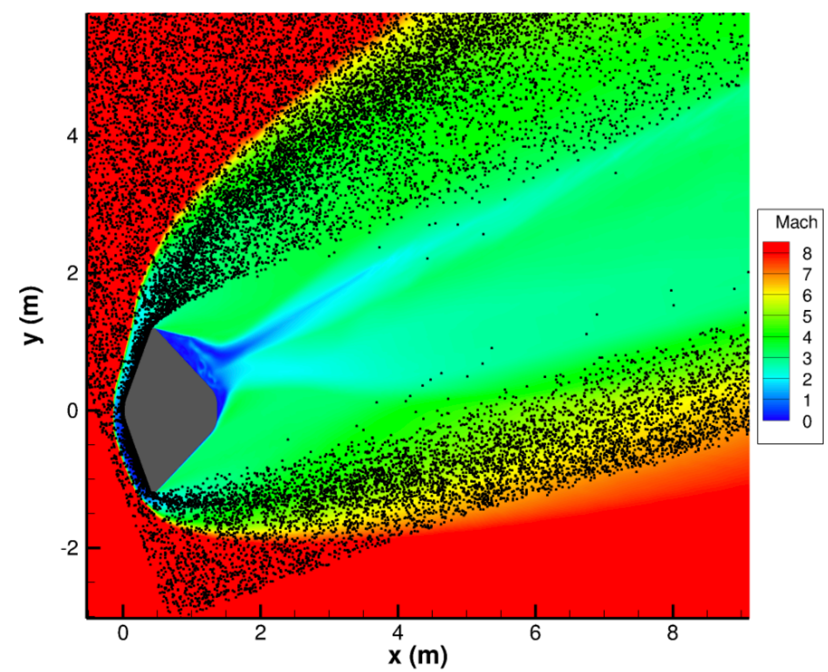
Particle-wall impacts

- Heating augmentation by collisional energy transfer
- Surface recession[2]

~300,000 3D hexahedral elements

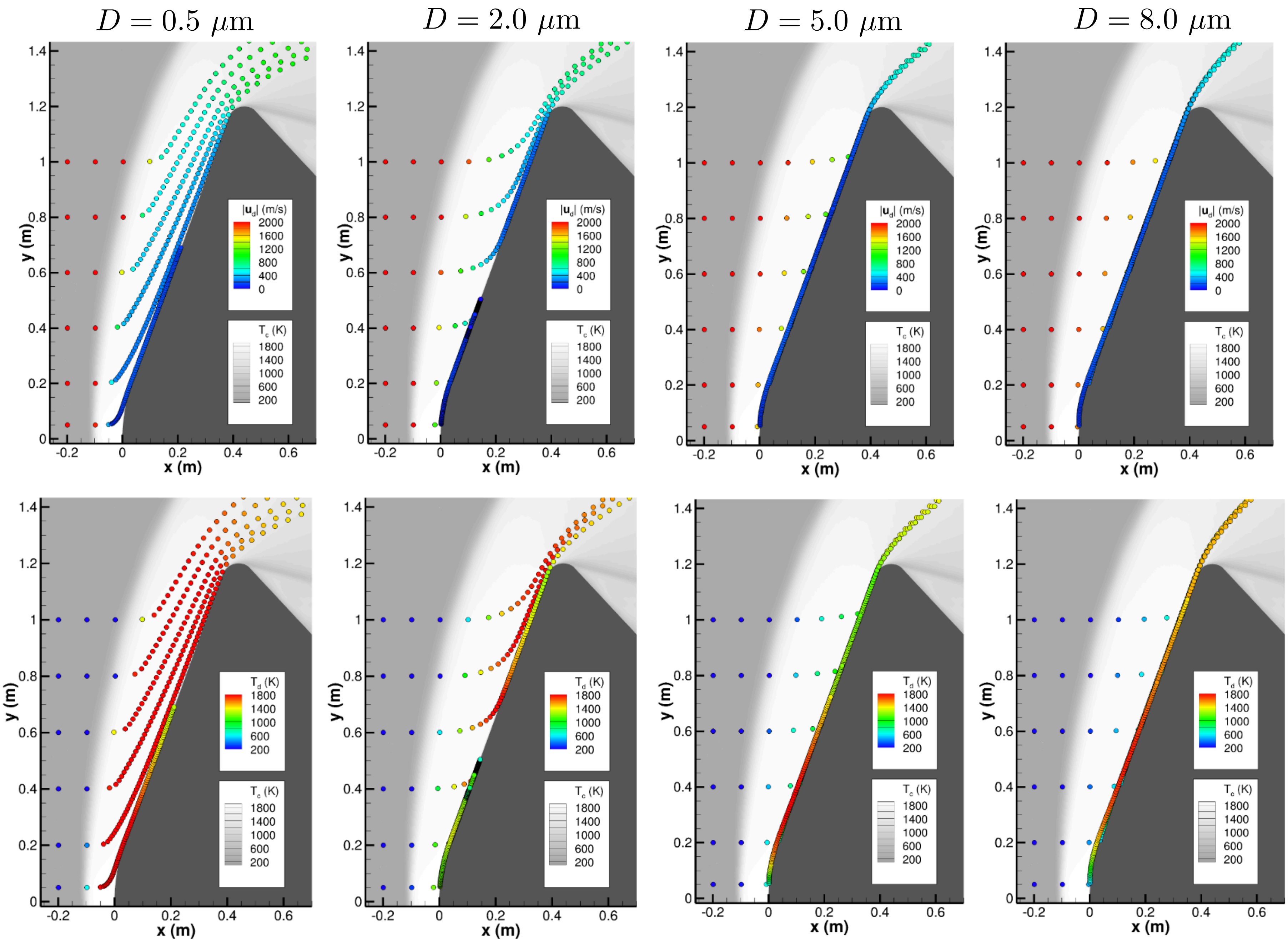
Third-order-accurate (p = 2) polynomials

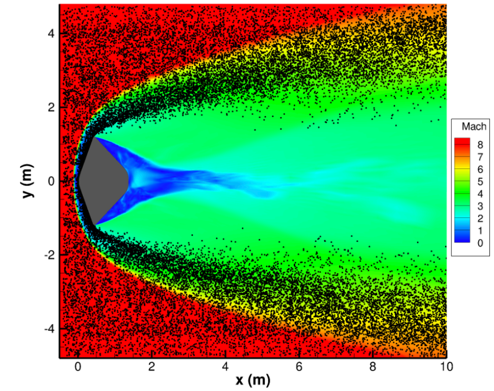




[1] Gulhan et al., *J Spacecraft Rockets*, 2019
[2] Palmer et al., *J Spacecraft Rockets*, 2020

Sample particle trajectories



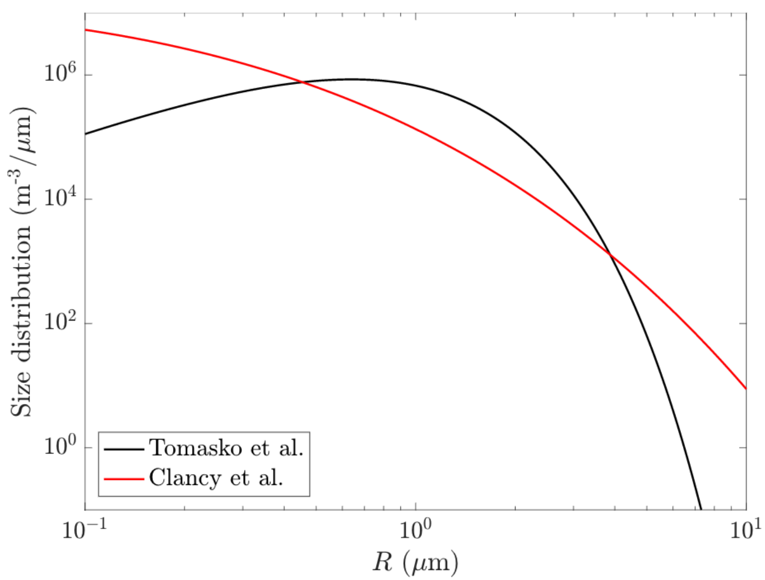


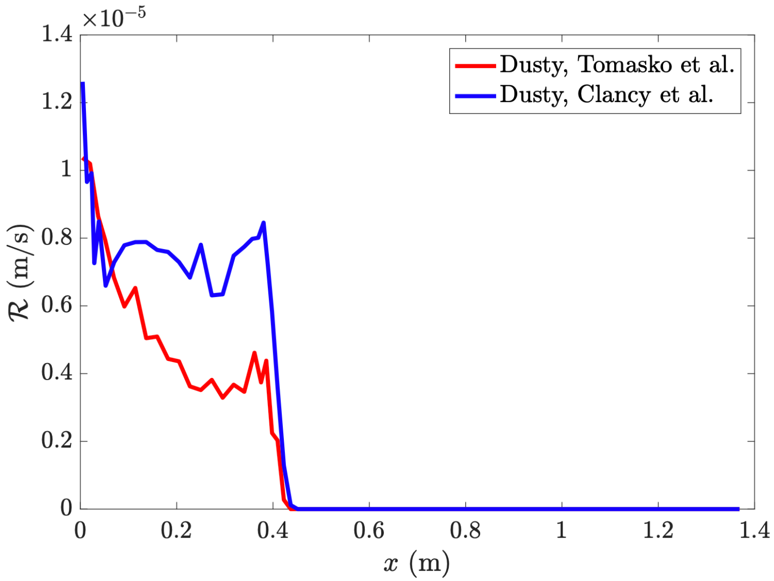
Effect of particle size distribution

Modified gamma distribution:

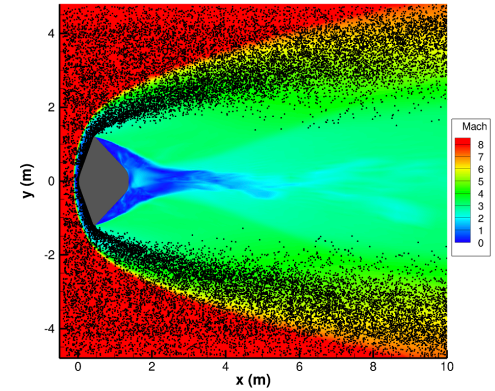
- Tomasko et al., *J Geophys Research: Planets*, 1999
- Clancy et al., *J Geophys Research: Planets*, 1995

Mass loading ratio (particle mass flux to gas mass flux): β = 0.01%



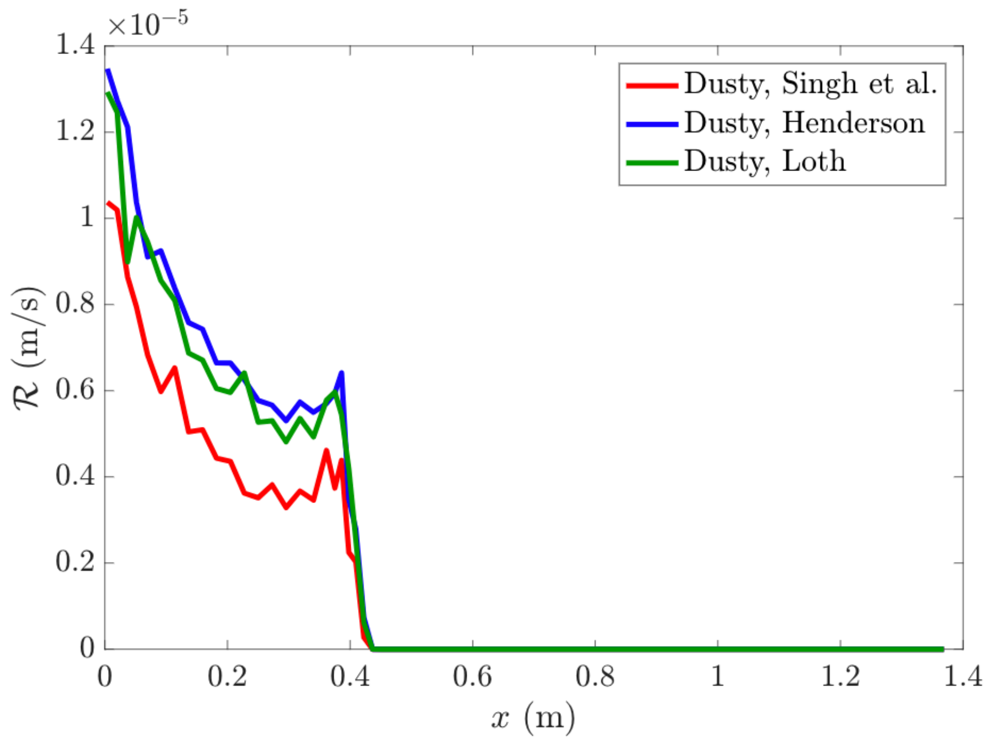


Effect of drag correlation



Drag correlations by:

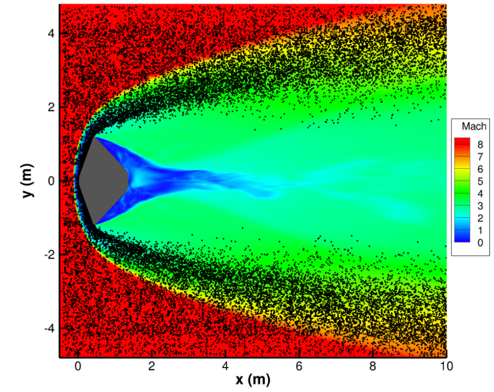
- Singh et al., https://arxiv.org/pdf/2012.04813
- Henderson, *AIAA J*, 1976
- Loth, *AIAA J*, 2008

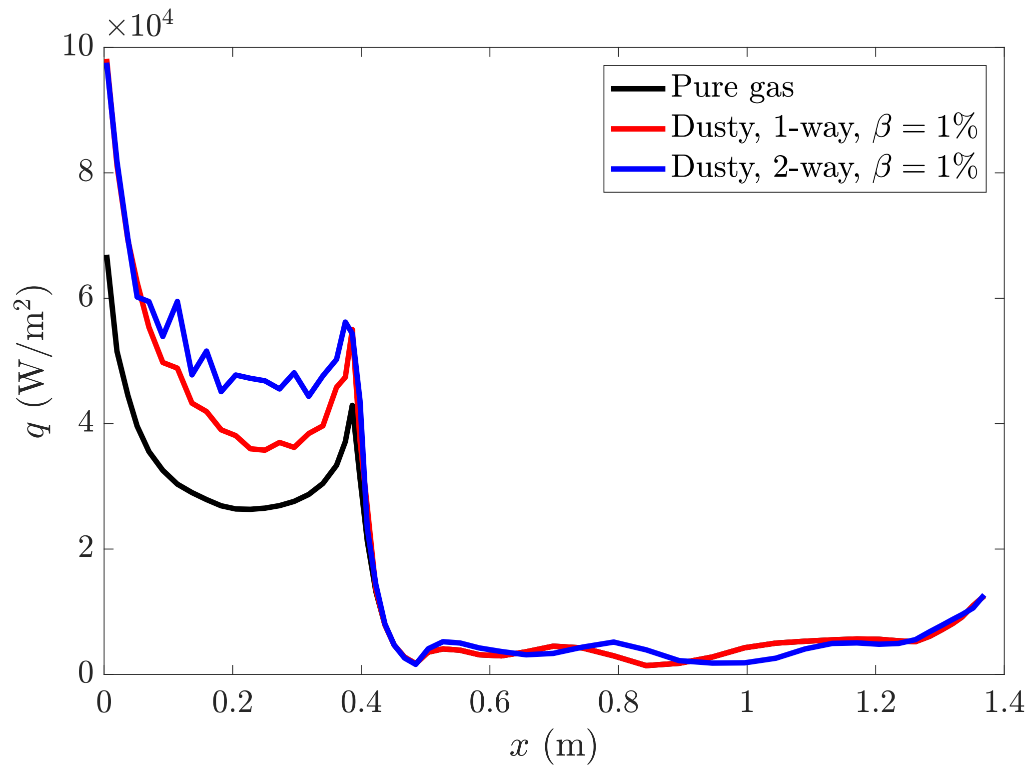


Effect of two-way coupling

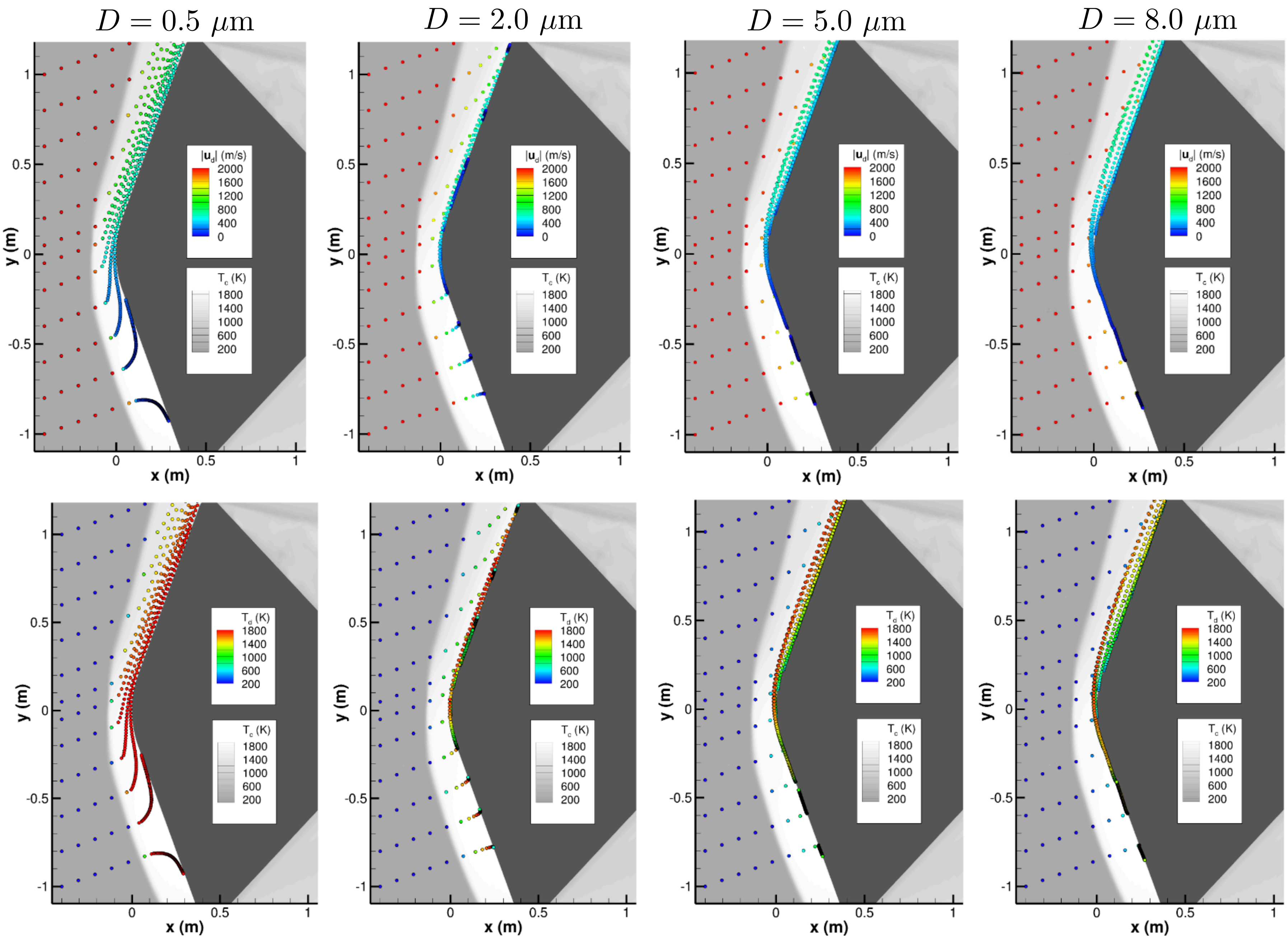
Particles also affect carrier gas

β = 1% mass loading ratio





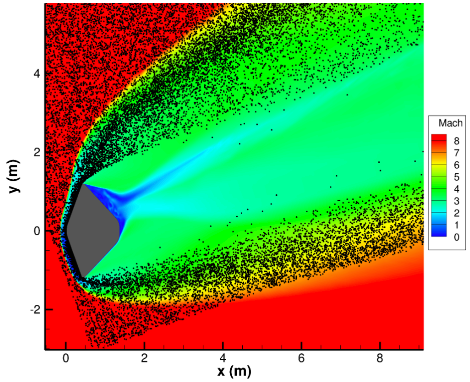
Sample particle trajectories (20 deg AoA)

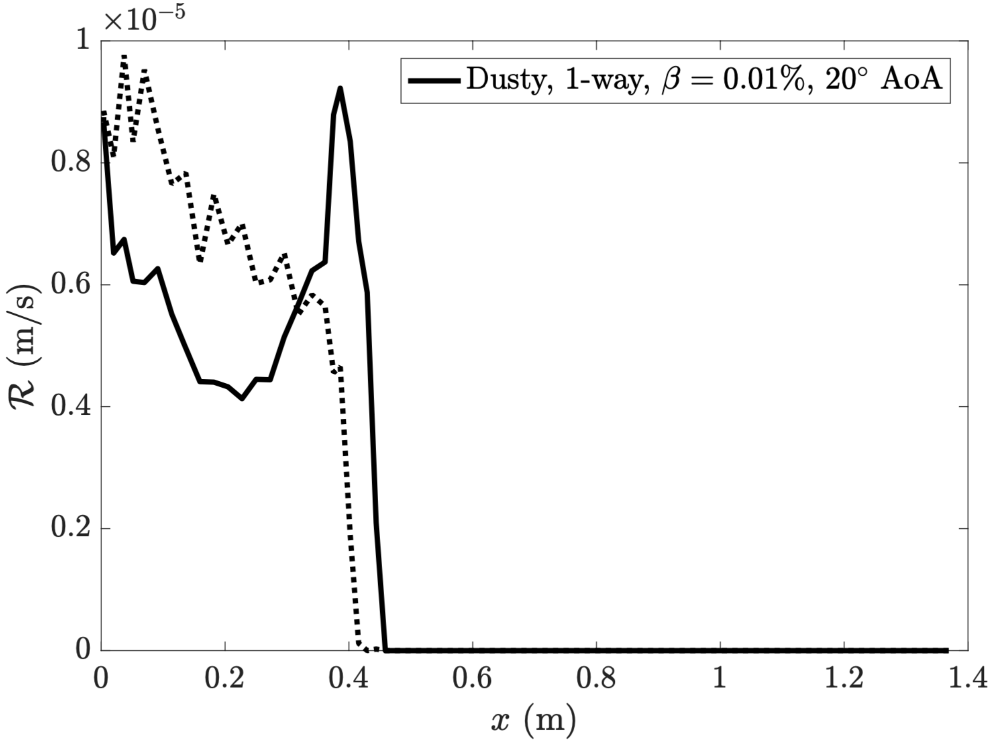


Surface recession

Solid line: windward side

Dotted line: leeward side





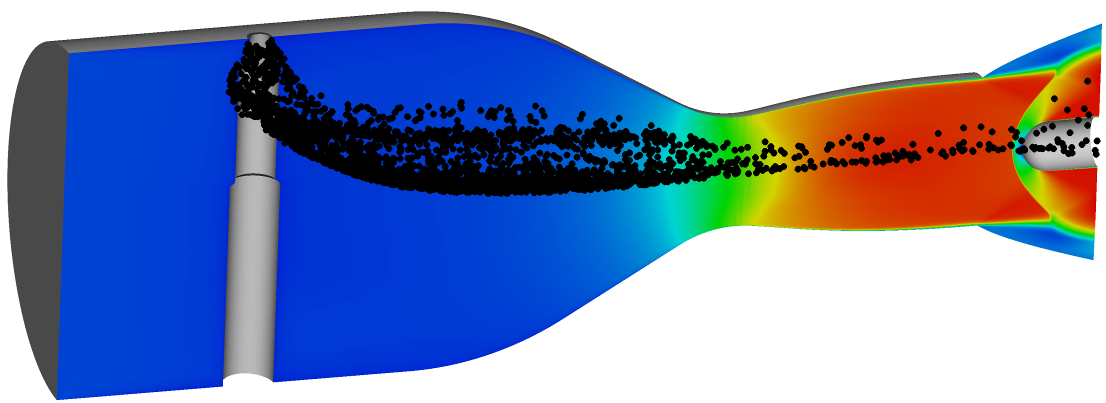
Main findings

- Dust impacts can cause surface erosion and heating augmentation
- Drag correlation by Singh et al. predicts less surface recession than popular correlations
- No direct interaction between particles and wake
- Large particles traverse the shock layer at higher speeds, lower temperatures than small particles
- Particle reverse-coupling at high dust loading can cause additional heating augmentation by transferring thermal energy to the boundary layer
- At nonzero angle of attack, leeward-side surface recession can be noticeably higher than windward-side recession

Summary & Outlook

Summary

- Developed a simple and robust shock capturing method for DG schemes
- Developed an Euler-Lagrange methodology in a DG framework suited for arbitrary curved, high-aspect-ratio elements
- Established significant benefits of the proposed DG methodology in the context of viscous hypersonic flows and particle-laden flows
- Applied the proposed methodology to hypersonic dusty flows over blunt bodies, with special application to Mars atmospheric entry



Outlook

- Further experimental validation: collaboration with DLR and NASA Ames
- Extend temperature range: thermochemical nonequilibrium

Thank you!

QUESTIONS